# Chapter 1

# Guo's Solution to Fall 2017

Who is Guo? Please turn to the last page to read my bio.

WA Q3 is the only problem where my solution is different from the SOA's.

$$\begin{cases} AL_x = YOS \times \alpha \times S_{fin} \times {}_{r-x}E_x \times \ddot{a}_r^{12} & \text{Mine} \\ AL_x = YOS \times \alpha \times S_{fin} \times v^{r-x} \times \ddot{a}_r^{12} & \text{SOA} \end{cases}$$

SOA assumes that the participant currently age $x$ will surely survive to the normal retirement age $r$, while I assume that she might live to $r$. Though you can argue either way, my solution is consistent with the AMLCR solution to Exercise 10.11.
If you solved this problem my way and failed the exam, I recommend that you write a formal letter to SOA explaining that you are using the AMLCR Exercise 10.11 approach and that you deserve credit.

MC Q1 ★★★★☆ Difficulty

$$\text{UDD:} \quad \mathring{e}_x = \frac{1}{2} + e_x$$

$$\begin{cases} e_{60} = {}_1p_{60} + {}_2p_{60} + {}_3p_{60} + \ldots = \dfrac{\ell_{61} + \ell_{62} + \ell_{63} + \ldots}{\ell_{60}} = 1 \\ e_{[58]+2} = \dfrac{\ell_{[58]+3} + \ell_{[58]+4} + \ell_{[58]+5} + \ldots}{\ell_{[58]+2}} = \dfrac{\ell_{61} + \ell_{62} + \ell_{63} + \ldots}{\ell_{[58]+2}} \end{cases}$$

$$\Rightarrow e_{[58]+2} = e_{60} \frac{\ell_{60}}{\ell_{[58]+2}} = 1 \times \frac{3904}{3548} = 1.1003382$$

$$\mathring{e}_{[58]+2} = 0.5 + 1.1003382 = 1.6003382 \quad \text{Answer: } B$$

MC Q2 ★★★☆☆ Difficulty

$$\ell_{53}^{(\tau)}(1 - 0.025 - 0.030) = 5000, \quad \ell_{53}^{(\tau)} = \frac{5000}{1 - 0.025 - 0.030} = 5291.0053$$

$$5000\left(1 - q_{54}^{(1)} - 0.040\right) = 4625, \quad 5000 q_{54}^{(1)} = 5000 \times (1 - 0.040) - 4625 = 175.0$$

$$ {}_2q_{53}^{(1)} = \frac{5291 \times 0.025 + 175}{5291} = 0.058075 \quad \text{Answer: } C$$

MC Q3 ★★★★☆ Difficulty
Let $S_t$ represent the state at time $t$. Let $Q$ represent the annual transition matrix.

$$S(0) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad S(1) = S(0)Q = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.64 & 0.16 & 0.02 \\ 0.36 & 0.24 & 0.40 \\ 0.00 & 0.00 & 1.00 \end{bmatrix} = \begin{bmatrix} 0.64 & 0.16 & 0.20 \end{bmatrix}$$

$$S(2) = S(1)Q = \begin{bmatrix} 0.64 & 0.16 & 0.20 \end{bmatrix} \begin{bmatrix} 0.64 & 0.16 & 0.02 \\ 0.36 & 0.24 & 0.40 \\ 0.00 & 0.00 & 1.00 \end{bmatrix} = \begin{bmatrix} 0.4672 & 0.1408 & 0.3920 \end{bmatrix}$$

Each healthy organism at $t = 0$ has $0.4672 + 0.1408 = 0.608$ chance of being alive at $t = 2$ and 0.392 chance of being dead at $t = 2$.

Imagine throwing a coin 1,000 times with each coin having $p = 0.608$ chance of getting a head and $q = 0.392$ chance of getting a tail. We need to find the probability of getting at least 625 heads. The binomial distribution $n = 1,000, p = 0.608$ is approximately normal with the following parameters:

$$\mu = np = 1000 \times 0.608 = 608, \quad \sigma^2 = npq = 1000 \times 0.608 \times 0.392 = 238.336$$

Since we treat $X$ as continuous without the continuity correction factor, we have:

$$P(X \geq 625) = 1 - P(X \leq 625) = 1 - \Phi\left(\frac{625-608}{\sqrt{238.336}}\right) = 1 - \Phi(1.1011693) = 1 - 0.86458851 = 0.13541149$$

Answer: $A$

I can see that some candidates will treat $X$ as discrete and do the following calculation:

$$P(X \geq 625) = 1 - P(X \leq 624) = 1 - \Phi\left(\frac{624-608}{\sqrt{238.336}}\right) = 1 - \Phi(1.0363946) = 1 - 0.84999096 = 0.15000904$$

However, this is probably not what SOA has intended.

MC Q4 ★★★★☆ Difficulty

$$\ddot{a}_{45}^{S} = 1 + p_{45}^{S} v \ddot{a}_{46}^{ILT}$$

$$p_{45}^{S} = \exp\left(-\int_0^1 \left(\mu_{45+t}^{ILT} + 0.05\right) dt\right) = \exp\left(-\int_0^1 \mu_{45+t}^{ILT} dt\right) e^{-0.05}$$

$$= p_{45}^{ILT} e^{-0.05} = e^{-0.05} \frac{\ell_{46}}{\ell_{45}}$$

$$\ddot{a}_{45}^{S} = 1 + e^{-0.05} v \frac{\ell_{46}}{\ell_{45}} \ddot{a}_{46}^{ILT} = 1 + e^{-0.05} \times 1.06^{-1} \times \frac{9,127,426}{9,164,051} \times 13.95459 = 13.472618$$

$$100 \times 13.472618 = 1,347.2618$$

Answer: $A$

MC Q5 ★★★★☆ Difficulty

$$A_{80}^{M} = v q_{80}^{M} + v p_{80}^{M} A_{81}^{ILT}$$

$$p_{80}^{M} = \exp\left(-0.8 \mu_{80+t}^{ILT} dt\right) = \left(-\exp\left(\mu_{80+t}^{ILT} dt\right)\right)^{0.8} = \left(p_{80}^{ILT}\right)^{0.8}$$

$$= \left(\frac{3,600,038}{3,914,365}\right)^{0.8} = 0.935226$$

$$A_{80}^{M} = \frac{1}{1.06} \times (1 - 0.935226) + \frac{1}{1.06} \times 0.935226 \times 0.6800019 = 0.66106553$$

$$100,000 \times 0.66106553 = 66,106.553$$

Answer: $B$

MC Q6 ★★★☆☆ Difficulty

$$E(Z) = {}_{25}p_x v^{25}, \quad E(Z^2) = {}_{25}p_x v^{50}$$

$$Var(Z) = {}_{25}p_x v^{50} - \left({}_{25}p_x v^{25}\right)^2 = {}_{25}p_x \left(1 - {}_{25}p_x\right) v^{50}$$

$$0.57(1-0.57)v^{50} = 0.1 \times 0.57 v^{25}, \quad (1+i)^{25} = 4.3, \quad i = 4.3^{1/25} - 1 = 6.008\%$$

Answer: $B$

MC Q7 ★★★★☆ Difficulty

$$P\ddot{a}_{55:\overline{10}|} = P(IA)_{\overset{1}{55}:\overline{10}|} + 300\,{}_{10}p_{55} v^{10} \ddot{a}_{65}$$

Note that ${}_{10}p_{55} v^{10} \ddot{a}_{65} = \ddot{a}_{55} - \ddot{a}_{55:\overline{10}|}$.

$$7.4575P = 0.51213P + 300(12.2758 - 7.4575), \quad P = 208.12282$$

Answer: $E$

MC Q8 ★★★★☆ Difficulty

$$G\ddot{a}_{70:\overline{10}|} = 0.7G + 0.05G\ddot{a}_{70:\overline{10}|} + 100,000\,{}_{10}E_{70} \ddot{a}_{80}$$

$$6.61842G = 0.7G + 0.05G \times 6.61842 + 100000 \times 0.330367 \times 5.90503$$

$$G = 34,914.137 \quad \text{Answer: } D$$

MC Q9 ★★★☆☆ Difficulty

$$P(1 + 0.95v^5) = 100,000 \times 0.95v^5 \left(0.02v + 0.98 \times 0.03v^2 + 0.98 \times 0.97 \times 0.04v^3\right)$$
$$P(1 + 0.95 \times 1.06^{-5}) = 100,000 \times 0.95 \times 1.06^{-5} \left(0.02 \times 1.06^{-1} + 0.98 \times 0.03 \times 1.06^{-2} + 0.98 \times 0.97 \times 0.04 \times 1.06^{-3}\right)$$

$$1.7098953P = 5463.3187, \quad P = 3195.12 \quad \text{Answer: } A$$

MC Q10 ★★★★☆ Difficulty

$$12(0.96G)\ddot{a}^{(12)}_{40:\overline{20|}} = 100,000\overline{A}_{40} + 200$$

$$\overline{A}_{40} = \frac{i}{\delta}A_{40}$$

$$\ddot{a}^{(12)}_{40:\overline{20|}} \approx \ddot{a}_{40:\overline{20|}} - \frac{11}{24}\left(1 - {}_{20}E_{40}\right)$$
$$= 11.76126 - \frac{11}{24}(1 - 0.294298) = 11.437813$$

$$12(0.96G)11.437813 = 100000 \times \frac{0.06}{\ln 1.06} \times 0.1613242$$

$$G = 126.07194 \quad \text{Answer: } C$$

Alternatively, under UDD, the following equation holds exactly:

$$\ddot{a}^{(12)}_{40:\overline{20|}} = \alpha(12)\ddot{a}_{40:\overline{20|}} - \beta(12)\left(1 - {}_{20}E_{40}\right)$$
$$= 1.00028 \times 11.76126 - 0.46812(1 - 0.294298) = 11.434200$$
$$12(0.96G)11.434200 = 100000 \times \frac{0.06}{\ln 1.06} \times 0.1613242$$
$$G = 126.11177$$

Generally, the two methods produce $\ddot{a}^{(m)}_{x:\overline{n|}}$ values that are very close.

MC Q11 ★★★★★ Difficulty

The PV random variable is

$$Z = \begin{cases} 100,000v^{T_{45}} & \text{if } T_{45} \le 20 \\ 0 & \text{if } T_{45} > 20 \end{cases}$$

$Z\left(T_{45}\right)$ is a decreasing function of $T_{45}$. The longer the insured (45) lives (e.g. the bigger $T_{45}$ is), the smaller the $Z$. A decreasing function reverses the percentile. So 95-th percentile of $Z$ maps to the 5-th percentile of $T_{45}$. We just need to calculate the 5-th percentile of $T_{45}$. Let $t^*$ represent the 5-th percentile of $T_{45}$.

$$P(T_{45} \le t^*) = 0.05, \quad \Rightarrow \quad P(T_{45} \ge t^*) = 0.95, \quad {}_{t^*}p_{45} = 0.95$$
$$\frac{\ell_{45+t^*}}{\ell_{45}} = \frac{\ell_{45+t^*}}{9,164,051} = 0.95, \quad \ell_{45+t^*} = 8,705,848.45$$

$$\ell_{54} = 8,712,621, \quad \ell_{55} = 8,640,861, \quad 54 < 45 + t^* < 55$$

Under UDD, what age has the population 8,705,848.45?

$$54 + \frac{\ell_{54} - 8,705,848.45}{\ell_{54} - \ell_{55}} = 54 + \frac{8,712,621 - 8,705,848.45}{8,712,621 - 8,640,861} = 54.094378$$

$$t^* = 54.094378 - 45 = 9.094378$$

$$Z_{95} = Z(T_{45} = 9.094378) = 100,000e^{-9.094378 \times 0.05} = 63,462.634 \quad \text{Answer: } C$$

MC Q12 ★★★☆☆ Difficulty

$$L_0 = 200,000v^{K_{45}+1} - \pi\ddot{a}_{\overline{K_{45}+1|}}$$

$\pi$ is proportional to 200,000 and so is $L_0$. To simplify calculation, we'll calculate $\sigma(L_0)$ for a unit death benefit and scale $\sigma(L_0)$ by the factor of 200,000 in the end.

$$L_0 = v^{K_{45}+1} - \pi \ddot{a}_{\overline{K_{45}+1|}} = v^{K_{45}+1} - \pi \frac{1 - v^{K_{45}+1}}{d} = -\frac{\pi}{d} + \left(1 + \frac{\pi}{d}\right) v^{K_{45}+1}$$

$$\pi = \frac{A_{45}}{\ddot{a}_{45}} = \frac{0.201202}{14.112092} = 0.014257$$

$$\sigma(L_0) = \left(1 + \frac{\pi}{d}\right) \sqrt{{}^2A_{45} - A_{45}^2} = \left(1 + \frac{0.014257}{1 - 1.06^{-1}}\right) \sqrt{0.068019 - 0.201202^2} = 0.20773845$$

$$200,000 \times 0.20773845 = 41,547.69 \quad \text{Answer: } A$$

MC Q13 ★★★★★ Difficulty
The net cash flow is proportional to two numbers:

- The number of policies, which is 100,000
- The death benefit, which is 1,000

To simplify thinking and calculation, we'll assume that the number of policies is one and the death benefit is also one. Then the expected cash flow is

$$q_{35} - p_{35} \times \frac{A_{35}}{\ddot{a}_{35}}$$

- For each insured alive at $t = 0$, $q_{35}$ number of people will die in Year 1. The death claim check in the amount of $q_{35} \times 1$ arrives on July 15, 2018.
- For each insured alive at $t = 0$, $p_{35}$ number of people will still be alive on July 15, 2018. For each of these survivors, a premium in the amount of $\frac{A_{35}}{\ddot{a}_{35}}$ is to be paid on July 15, 2018

The net cash flow per insured at $t = 0$ per 1 dollar death benefit is:

$$q_{35} - p_{35} \times \frac{A_{35}}{\ddot{a}_{35}} = 0.002014 - 0.997986 \times \frac{0.128719}{15.392624} = -0.00633154$$

$$100,000 \times 1,000(-0.00633154) = -633,154 \quad \text{Answer: } A$$

This is a surprise problem unseen in the past. Though the calculation is simple, it may take you a while to think through. Hence the difficulty level is 5.

MC Q14 ★★★☆☆ Difficulty

$${}_1V = 1,000,000 \left(1 - \frac{\ddot{a}_{41}}{\ddot{a}_{40}}\right) = XA_{41}$$

$$1,000,000 \left(1 - \frac{14.68645}{14.81661}\right) = 0.16869X$$

$$X = 52,076.208 \quad \text{Answer: } D$$

MC Q15 ★★★★☆ Difficulty

$$\begin{cases} P\ddot{a}_{45} + {}_{20}E_{45} W \ddot{a}_{65} = 1,000A_{45} \\ {}_{20}V + (P + W)\ddot{a}_{65} = 1,000A_{65} \end{cases}$$

$$\begin{cases} 14.11209P + 0.256341W \times 9.89693 = 1,000 \times 0.2012023 \\ 0 + (P + W)9.89693 = 1,000 \times 0.4397965 \end{cases}$$

$$P = 7.6426482, \quad W = 36.795021, \quad \text{Answer: } D$$

MC Q16 ★★★★☆ Difficulty
You can do complex math if you like. However, I'm going to use a shortcut, which can be found in MLC study manual.

> For a fully discrete whole life insurance, if the Year 1 acquisition cost is $\alpha$ and Year 2+ maintenance cost is $\beta$ where $\beta \leq \alpha$, then the expense policy value at $t$ is:
>
> $${}_tV^e = -(\alpha - \beta) \frac{\ddot{a}_{x+t}}{\ddot{a}_x}$$

Before explaining this formula, I want to caution you that this formula doesn't work if there's death claim expense. If there's claim cost, you have to adjust the shortcut formula as explained in my study guide.

For this problem, the claim cost is zero and the shortcut always works.

To understand this formula, note that the expense policy value is caused by Year 1 extra cost $\alpha - \beta$. If $\alpha - \beta = 0$, then the expense policy value will always be zero; the gross and net policy values will always be equal.

The Year 1 extra cost is funded by level premiums $\dfrac{(\alpha-\beta)}{\ddot{a}_x}$. However, the renewal extra cost is zero. Hence

$$ {}_tV^e = \text{PV future extra cost} - \text{PV future premiums to fund Yr 1 extra cost} = 0 - \frac{(\alpha-\beta)}{\ddot{a}_x}\ddot{a}_{x+t} $$

$$ {}_{10}V^g = {}_{10}V^n + {}_{10}V^n = 2290 - (30-5)\times\frac{\ddot{a}_{x+10}}{\ddot{a}_x} = 2290 - (30-5)\times\frac{11.4}{14.8} = 2270.7432 $$

Answer: $E$

MC Q17 ★★★★☆ Difficulty

We need to set up systems of equations twice, the first time to ignore the corridor and the second time to consider the corridor.

First, we ignore the corridor:

$$ \begin{cases} AV_6 = (200,000 + 25,000\times 0.98 - COI)1.05 \\ COI = (500,000 - AV_6)\times\dfrac{30}{1000}\times\dfrac{1}{1.05} \end{cases} $$

$$ \Rightarrow AV_6 = 227,551.55, \quad COI = 7,784.2415 $$

Second, we consider the corridor:

$$ \begin{cases} AV_6 = (200,000 + 25,000\times 0.98 - COI)1.05 \\ COI = (2.5-1)AV_6\times\dfrac{30}{1000}\times\dfrac{1}{1.05} \end{cases} $$

$$ \Rightarrow AV_6 = 225,574.16, \quad COI = 9,667.4641 $$

Finally, we choose the maximum ADB and the maximum COI.

$$ \begin{cases} ADB = \max\left(500000 - 227551.55, \quad (2.5-1)225574.16\right) = \max(272448.45, \quad 338361.0) = 338,361 \\ COI = \max(7784.2415, \quad 9667.4641) = 9,667.4641 \end{cases} $$

Answer: $E$

MC Q18 ★★★★☆ Difficulty

If the insured is still alive at $t=10$, he has a fully paid-up whole life insurance on (35). The expected future loss at $t=10$ is equal to the single net premium of the fully paid up whole life policy on (35).

$$ \Delta\left[E\left(L_{10}\right)\right] = 10,000\left(A_{35}^{i=0} - A_{35}^{i=0.06}\right) = 10,000(1-0.1287194) = 8,712.806 $$

Answer: $E$

MC Q19 ★★★★☆ Difficulty

The fundamental theorem of calculus:

$$ \frac{d}{dx}\int_a^x f(t)dt = f(x) $$

$$ \left(\overline{Ia}\right)_{x:\overline{t}|} = \int_0^t s\,{}_sp_x\,v^s ds, \qquad \frac{\mathrm{d}\left(\overline{Ia}\right)_{x:\overline{t}|}}{\mathrm{d}t} = t\,{}_tp_x\,v^t $$

$$ \left[\frac{\mathrm{d}\left(\overline{Ia}\right)_{40:\overline{t}|}}{\mathrm{d}t}\right]_{t=10.5} = 10.5\times{}_{10.5}p_{40}\times 1.06^{-10.5} $$

$$ {}_{10.5}p_{40} = \frac{\ell_{50.5}}{\ell_{40}} = 0.5\times\frac{\ell_{50}+\ell_{51}}{\ell_{40}} = 0.5\times\frac{8,950,901 + 8,897,913}{9,313,166} = 0.95825705 $$

$$ \Rightarrow \left[\frac{\mathrm{d}\left(\overline{Ia}\right)_{40:\overline{t}|}}{\mathrm{d}t}\right]_{t=10.5} = 10.5\times 0.95825705\times 1.06^{-10.5} = 5.4570727 $$

Answer: $C$

MC Q20 ★★★☆☆ Difficulty

There's no mid-year exit benefit. Hence the normal cost is the APV of the pension benefit to be earned due to the next year's service.

$$ NC = S_{64}\times 1\text{ YOS}\times 1.5\%\times{}_{20}p_{45}^{(\tau)}\times v^{20}\times\ddot{a}_{65} $$

$$ \text{Expected salary from age 64 to 65 } = \text{salary from age 44 to 45}\times 1.04^{20} = 120,000\times 1.04^{20} $$

$$ NC = 120,000\times 1.04^{20}\times 1\times 0.015\times 0.552\times 1.05^{-20}\times 10.6 = 8,697.5764 $$

Answer: $E$

Note that 15 years of past service is irrelevant to the normal cost.

WA Q1 (*a*) ★★★★☆ Difficulty

$$\begin{cases} P^g \ddot{a}_{50:\overline{10}|} = (450 + 0.36P^g) + (50 + 0.04P^g)\ddot{a}_{50:\overline{10}|} + 100,000A_{50:\overline{20}|} \\ P^n \ddot{a}_{50:\overline{10}|} = 100,000A_{50:\overline{20}|} \end{cases}$$

$$\begin{cases} 7.57371P^g = (450 + 0.36P^g) + (50 + 0.04P^g)7.57371 + 100,000 \times 0.360839 \\ 7.57371P^n = 100,000 \times 0.360839 \end{cases}$$

$$P^g = 5,341.32, \quad P^n = 4,764.36, \quad P^e = P^g - P^n = 576.96$$

(*b*) ★★★★☆ Difficulty

$$\begin{aligned} {}_1V^e &= \text{APV future expenses as of } t = 1 - \text{APV future expense premiums as of } t = 1 \\ &= \left(50 + 0.04P^g\right)\ddot{a}_{51:\overline{9}|} - P^e \ddot{a}_{51:\overline{9}|} \\ &= (50 + 0.04 \times 5,341.32)7.00963 - 576.96 \times 7.00963 = -2,196.17 \end{aligned}$$

Alternatively, you can calculate ${}_1V^e = {}_1V^g - {}_1V^n$ but this is more work.

(*c*) ★★★★☆ Difficulty

$P^e$ is greater than the renewal expense because $P^e$ not only funds the renewal expense but also the higher year 1 acquisition cost. Hence at the end of Year 1, the APV of future expense premiums will be greater than the APV of the future renewal expenses, generating a negative expense premium reserve. However, ${}_tV^e = 0$ for $t = 10, 11, 12, \ldots, 20$ because there will be no renewal expenses or renewal expense premiums beyond the first 10 years.

(*d*) ★★★★☆ Difficulty

Under FPT, for the purpose of reserving, the original policy is replaced by two brand new policies.

- Policy A: a 1-year term policy, and
- Policy B: if the insured is still alive at the end of Year 1, a 9-pay 19-year endowment policy.

Policy A has a zero reserve at the end of Year 1.

$${}_1V^{FPT} = 0$$

To calculate Policy B's reserve, we need to find its net premium.

$$\beta = 100,000\frac{A_{51:\overline{19}|}}{\ddot{a}_{51:\overline{9}|}} = 100,000 \times \frac{0.378812}{7.00963} = 5,404.17$$

$${}_2V^{FPT} = 100,000A_{52:\overline{18}|} - \beta\ddot{a}_{52:\overline{8}|} = 100,000 \times 0.397673 - 5404.17 \times 6.41138 = 5,119.11$$

After 10 years, the policy is fully paid up and there are no more premiums. For $t = 10, 11, \ldots, 20$, the gross premium reserve, the net premium reserve, and the FPT reserve are all equal to the net single premium of the fully paid-up endowment policy.

$${}_{19}V^g = {}_{19}V^n = {}_{19}V^{FPT} = 100,000A_{69:\overline{1}|} = 100,000 \times 1.06^{-1} = 94,339.62$$

(*e*) ★★★★☆ Difficulty

- FPT is an improved reserving method over the net premium reserve method. It keeps the good thing about the net premium reserve (e.g. computation simplicity) but addresses the major draw back of the net premium reserve.
- Net premium reserve is computationally simple because it ignore expenses. However, it creates higher reserves than the gross premium policy values, requiring insurers to hold higher capital than necessary.
- Under the net premium reserving method, the negative expense policy values (which are assets) are ignored.
- FPT attempts to address the major draw back of the net premium reserve. Under FPT, year 1 reserve is zero, reducing the capital strain caused by the net premium reserve method for early years
- FPT reserve is often somewhere between the gross premium reserve and the net premium reserve
- FPT also ignores expenses and is computationally simple

WA Q2

(*a*) ★★★☆☆ Difficulty

$$\begin{aligned} AV_3 &= (AV_2 + P_3 - EC_3 - CoI_3)(1 + i) \\ &= \left(8166 + 4,000 - 4,000 \times 0.05 - 150,000 \times \frac{2.62}{1,000 \times 1.04}\right)1.04 = 12,824 \end{aligned}$$

(*b*) ★★★★★ Difficulty

| Year $t$ | $AV_{t-1}$ | $P_t$ | $E_t$ | $I_t$ | $EDB_t$ | $ESB_t$ | $EAV_t$ | $Pr_t$ | $\Pi_t$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 800 | | 0.00 | 0.00 | 0.00 | −800.00 | −800.00 |
| 1 | 0 | 4,800 | 1,200 | 288.00 | 231.22 | 114.53 | 3,277.38 | 264.88 | 264.88 |
| 2 | 3,647 | 4,800 | 336 | 648.88 | 238.00 | 1,331.20 | 6,523.00 | 667.68 | 600.01 |
| 3 | 8,166 | 4,800 | 336 | 1,010.40 | 245.01 | 12,822.74 | 0.00 | 572.65 | 411.07 |

| Year $t$ | $p^d_{35+t-1}$ | $p^w_{35+t-1}$ | $p^{00}_{35+t-1} = (1-p^d_{35+t-1})(1-p^w_{35+t-1})$ |
|---|---|---|---|
| 0 | | | |
| 1 | 0.0015 | 0.1 | 0.89865 |
| 2 | 0.0015 | 0.2 | 0.7988 |
| 3 | 0.0015 | 1.0 | 0 |

| Year $t$ | $AV_t$ | $SurrChrg_t$ | $CV_t$ |
|---|---|---|---|
| 1 | 3,647 | 2,500 | 1,147 |
| 2 | 8,166 | 1,500 | 6,666 |
| 3 | 12,824 | 0 | 12,842 |

At $t = 0$: $Pr_0 = \Pi_0 = -800$
First Year:

- Premium $P_1 = 4,800$
- Expense: $E_1 = 4,800 \times 0.25 = \frac{4,800}{4} = 1,200$
- Interest earned: $I_1 = (4,800 - 1,200)0.08 = 288$
- Expected death benefit $EDB_1 = 0.0015(150,000 + 3,647 + 500) = 231.22$
- Expected surrender benefit $ESB_1 = 1,147(1 - 0.0015)0.1 = 114.53$.
- Expected account value $EAV_1 = (1 - 0.0015)(1 - 0.1)3,647 = 3,277.38$
- $Pr_1 = 4,800 - 1,200 + 288 - 231.22 - 114.53 - 3,277.38 = 264.88$
- $\Pi_1 = 264.88$

Second Year:

- Premium $P_2 = 4,800$
- Expense: $E_2 = 4,800 \times 0.07 = 336$
- Interest earned: $I_2 = (3,647 + 4,800 - 336)0.08 = 648.88$
- Expected death benefit $EDB_2 = 0.0015(150,000 + 8,166 + 500) = 238$
- Expected surrender benefit $ESB_2 = 6,666(1 - 0.0015)0.2 = 1,331.20$
- Expected account value $EAV_2 = (1 - 0.0015)(1 - 0.2)8,166 = 6,523.000$
- $Pr_2 = 3,647 + 4,800 - 336 + 648.88 - 238 - 1,331.20 - 6,523 = 667.68$
- $\Pi_2 = 667.68(1 - 0.0015)(1 - 0.1) = 600.01$

Third Year:

- Premium $P_3 = 4,800$
- Expense: $E_3 = 4,800 \times 0.07 = 336$
- Interest earned: $I_3 = (8,166 + 4,800 - 336)0.08 = 1,010.40$
- Expected death benefit $EDB_3 = 0.0015(150,000 + 12,842 + 500) = 245.01$
- Expected surrender benefit $ESB_3 = 12,842(1 - 0.0015)1 = 12,822.74$
- Expected account value $EAV_3 = (1 - 0.0015)(1 - 1)12,842 = 0$
- $Pr_3 = 8,166 + 4,800 - 336 + 1,010.40 - 245.01 - 12,822.74 - 0 = 572.65$
- $\Pi_3 = 572.65(1 - 0.0015)(1 - 0.1)(1 - 0.0015)(1 - 0.2) = 411.07$

(*c*) ★★★☆☆ Difficulty

$$NPV = -800 + \frac{264.88}{1.1} + \frac{600.01}{1.1^2} + \frac{411.07}{1.1^3} = 245.52$$

(*d*) ★★★☆☆ Difficulty
EPV of the premiums at the discount rate is:

$$4800\left(1 + \frac{(1-0.0015)(1-0.1)}{1.1} + \frac{(1-0.0015)^2(1-0.1)(1-0.2)}{1.1^2}\right) = 11,569.02$$

The profit margin is NPV divided by EPV of the premiums:

$$\frac{245.52}{11,569.02} = 2.12\%$$

This problem is conceptually simple but it has tedious calculations that are prone to errors.

WA Q3

($a$) ★★★★☆ Difficulty

The most important thing you need to know is that the actuarial assumptions (such as 2% salary growth) apply to part ($b$) and ($c$), not ($a$). Perhaps SOA should have made this point clear by, for example, moving the assumptions to the next page. This will alert people not to use the 2% salary growth factor to calculate the accrued pension benefit. After all, under PUC, we do project salary growth when calculating the accrued pension benefit.

- DOB: 1/1/1958
- Plan entry date: 1/1/1988. Entry age $e = 30$
- Valuation date: 12/31/2017. We change it to 1/1/2018. Attained age at valuation date $x = 60$
- YOS: $x - e = 30$
- Retirement age: $r = 65$
- $\alpha = 0.02$

To be conservative, we'll the past 3 years' salaries instead of worrying about salary growth.

$$B_x = 30 \times 0.02 \times \frac{95,000 + 92,000 + 90,000}{3} = 55,400$$

($b$) ★★★★★ Difficulty

TUC: no projection of salary growth

$$S_{fin} = \frac{95000 + 92000 + 90000}{3} = 92333.333$$

For normal retirement age $r$,

$$AL_x = YOS \times \alpha \times S_{fin} \times {}_{r-x}E_x \times \ddot{a}_r^{12}$$

We'll need to consider early retirement.

$$AL_x = 0.5(AL_x \text{ if retire at age 65 }) + 0.5(AL_x \text{ if retire at age 60})$$

$$AL_x \text{ if retire at age 65}$$
$$= 30 \times 0.02 \times 92333.333 \times {}_5E_{60} \times \ddot{a}_{65}^{(12)}$$
$$= 30 \times 0.02 \times 92333.333 \times 0.687563 \times \left(9.89693 - \frac{11}{24}\right) = 359525.49$$

$$AL_x \text{ if retire at age 60}$$
$$= 30 \times 0.02 \times 92333.333 \times \big(1 - (65 - 60)12 \times 0.005\big)\, \ddot{a}_{60}^{(12)}$$
$$= 30 \times 0.02 \times 92333.333 \times \big(1 - (65 - 60)12 \times 0.005\big) \times \left(11.14535 - \frac{11}{24}\right) = 414442.5$$

$$AL_x = 0.5(359525.49 + 414442.5) = 386984.00$$

If she indeed retires at age 60, then $AL_x = 414442.5$. The increase of the actuarial liability is:

$$414442.5 - 386984.00 = 27458.5 \quad \text{Loss}$$

($c$) ★★★★★ Difficulty

PUC: project salary growth

- Salary from 1/1/2018 to 1/1/2019 (from age 60 to 61) is $95,000 \times 1.02$
- Salary for final 3 years for normal retirement age: $64 - 65$ is $95,000 \times 1.02^5$, $63 - 64$ is $95,000 \times 1.02^4$, $62 - 63$ is $95,000 \times 1.02^3$.

We'll need to consider early retirement.

$$AL_x = 0.5(AL_x \text{ if retire at age 65 }) + 0.5(AL_x \text{ if retire at age 60})$$

$$AL_x \text{ if retire at age 65}$$
$$= 30 \times 0.02 \times 95000 \times \frac{1.02^5 + 1.02^4 + 1.02^3}{3} {}_5E_{60} \times \ddot{a}_{65}^{(12)}$$
$$= 30 \times 0.02 \times 95000 \times \frac{1.02^5 + 1.02^4 + 1.02^3}{3} \times 0.687563 \times \left(9.89693 - \frac{11}{24}\right) = 400453.63$$

$$AL_x \text{ if retire at age 60}$$
$$= 30 \times 0.02 \times \frac{95000+92000+90000}{3} \times (1-(65-60)12 \times 0.005)\ddot{a}_{60}^{(12)}$$
$$= 30 \times 0.02 \times \frac{95000+92000+90000}{3} \times (1-(65-60)12 \times 0.005) \times \left(11.14535 - \frac{11}{24}\right) = 414442.50$$

$$AL_x = 0.5(400453.63 + 414442.50) = 407448.07$$

If she indeed retires at age 60, then $AL_x = 414442.5$. The increase of the actuarial liability is:

$$414442.5 - 407448.07 = 6994.43 \quad \text{Loss}$$

This problem is conceptually simple but it has tedious calculations that are prone to errors.

WA Q4

(*a*) ★★★☆☆ Difficulty

$$L_5 = 10,000v^{K_{40}+1} - P\ddot{a}_{\overline{\min(K_{40}+1,5)}|}$$

(*b*) ★★★☆☆ Difficulty

$$_5V = 10,000A_{40} - P\ddot{a}_{40:\overline{5}|} = 10000 \times 0.161324 - 166.58 \times 4.44007 = 873.61$$

(*c*) ★★★★☆ Difficulty

$$Z = 10,000v^{K_{40}+1}$$
$$\Rightarrow \quad \sigma_Z = 10,000\sqrt{{}^2A_{40} - A_{40}^2} = 10000\sqrt{0.048633 - 0.161324^2} = 1503.5813$$

$$Y = P\ddot{a}_{\overline{\min(K_{40}+1,5)}|} = P \times \frac{1-v^{\min(K_{40}+1,5)}}{d}$$
$$\sigma_Y = \frac{P}{d}\sqrt{{}^2A_{40:\overline{5}|} - A_{40:\overline{5}|}^2}$$

$$A_{40:\overline{5}|} = A_{\overset{1}{40}:\overline{5}|} + {}_5E_{40} = A_{40} - {}_5E_{40}A_{45} + {}_5E_{40} = A_{40} + (1-A_{45})\,{}_5E_{40}$$
$$= 0.161324 + (1-0.20120)0.735294 = 0.748675$$

$${}^2A_{40:\overline{5}|} = {}^2A_{40} + \left(1 - {}^2A_{45}\right){}_5E_{40}v^5 = 0.048633 + (1-0.068019)0.735294 \times 1.06^{-5} = 0.560714$$

$$\Rightarrow \quad \sigma_Y = \frac{P}{d}\sqrt{{}^2A_{40:\overline{5}|} - A_{40:\overline{5}|}^2} = \frac{166.58}{1-1.06^{-1}}\sqrt{0.560714 - 0.748675^2} = 41.59$$

(*d*) ★★★★★ Difficulty

$$Cov(Y,Z) = E(YZ) - E(Y)E(Z)$$

$$Y = \begin{cases} P\ddot{a}_{\overline{K_{40}+1}|} = P\dfrac{1-v^{K_{40}+1}}{d} & \text{if } K_{40}+1 \le 5 \\ P\ddot{a}_5 & \text{if } K_{40}+1 > 5 \end{cases}$$

$$Z = 10,000v^{K_{40}+1}$$

$$\Rightarrow \quad YZ = 10,000P \times \begin{cases} \dfrac{v^{K_{40}+1} - \left(v^2\right)^{K_{40}+1}}{d} & \text{if } K_{40}+1 \le 5 \\ \ddot{a}_5 v^{K_{40}+1} & \text{if } K_{40}+1 > 5 \end{cases}$$

$$E(YZ) = 10,000P\left(\frac{A_{\overset{1}{40}:\overline{5}|} - {}^2A_{\overset{1}{40}:\overline{5}|}}{d} + \ddot{a}_5\,{}_5E_{40}\,A_{45}\right)$$

$$= 10000 \times 166.58\left(\frac{0.013381 - 0.011260}{1-1.06^{-1}} + 4.465106 \times 0.735294 \times 0.20120\right) = 1162801.6$$

$$E(Y) = P\ddot{a}_{40:\overline{5}|} = 166.58 \times 4.44007 = 739.62686$$
$$E(Z) = 10000A_{40} = 10000 \times 0.161324 = 1613.24$$

$$\Rightarrow \quad Cov(Y,Z) = 1162801.6 - 739.62686 \times 1613.24 = -30,394.036$$

$(e)$ ★★★★☆ Difficulty

$$L_5 = Z - Y$$

$$Var(L_5) = Var(Z) + Var(Y) - 2Cov(Y, Z) = 1503.5813^2 + 41.59^2 - 2(-30,394.036) = 2,323,274.5$$

$$\sigma(L_5) = \sqrt{2,323,274.5} = 1,524.2291$$

$(f)$ ★★★☆☆ Difficulty $Cov(Y, Z) < 0$ because $Y$ and $Z$ goes in opposite directions. As the insured lives longer, the PV of the death benefit gets smaller, yet the PV of the premiums increases and then becomes a constant.

WA Q5

$(a)$ ★★★★☆ Difficulty

$$\frac{\mathrm{d}}{\mathrm{d}t} {}_tp_{xy}^{01} = {}_tp_{xy}^{00} \mu_{x+t:y+t}^{01} - {}_tp_{xy}^{01} \mu_{x+t:y+t}^{13}$$

$${}_tp_{xy}^{00} = e^{-(0.01+0.001+0.014)t} = e^{-0.025t}$$

$$\Rightarrow \quad \frac{\mathrm{d}}{\mathrm{d}t} {}_tp_{xy}^{01} = 0.01e^{-0.025t} - 0.012\, {}_tp_{xy}^{01} \tag{1.1}$$

SOA gives us the solution to Equation 1.1:

$${}_tp_{xy}^{01} = \frac{0.010}{0.013} \left( e^{-0.012t} - e^{-0.025t} \right) \tag{1.2}$$

We just need to verify that Solution 1.2 satisfies Equation 1.1 and following the boundary condition

$$\left[ {}_tp_{xy}^{01} \right]_{t=0} = 0 \tag{1.3}$$

$$\left[ {}_tp_{xy}^{01} \right]_{t=0} = \frac{0.010}{0.013} \left( e^{-0.012\times 0} - e^{-0.025\times 0} \right) = 0$$

The boundary condition 1.3 is met.

$$\frac{\mathrm{d}}{\mathrm{d}t} {}_tp_{xy}^{01} = \frac{0.010}{0.013} \left( -0.012e^{-0.012t} + 0.025e^{-0.025t} \right)$$

$$0.01e^{-0.025t} - 0.012\, {}_tp_{xy}^{01} = 0.01e^{-0.025t} - 0.012 \times \frac{0.010}{0.013} \left( e^{-0.012t} - e^{-0.025t} \right)$$

$$= \frac{0.010}{0.013} \left( -0.012e^{-0.012t} + (0.012 + 0.013)e^{-0.025t} \right)$$

$$= \frac{0.010}{0.013} \left( -0.012e^{-0.012t} + 0.025e^{-0.025t} \right)$$

Equation 1.1 is also met. Hence Solution 1.2 is correct.

$(b)(i)$ ★★★☆☆ Difficulty

EPV of the death benefit triggered by path $0 \to 3$ is:

$$100,000 \int_0^{10} {}_tp_{xy}^{00} \mu_{x+t:y+t}^{03} e^{-0.06t} dt$$

$$= 100,000 \int_0^{10} 0.001e^{-0.025t} e^{-0.06t} dt = 100 \int_0^{10} e^{-0.085t} dt = 100 \times \frac{1 - e^{-0.085\times 10}}{0.085} = 673.63$$

$(b)(ii)$ ★★★★☆ Difficulty

EPV of the death benefit triggered by path $0 \to 1 \to 3$ is:

$$100,000 \int_0^{10} {}_tp_{xy}^{01} \mu_{x+t:y+t}^{13} e^{-0.06t} dt$$

$$= 100,000 \int_0^{10} \frac{0.010}{0.013} \left( e^{-0.012t} - e^{-0.025t} \right) 0.012e^{-0.06t} dt = 361.98$$

$(c)$ ★★★★☆ Difficulty

$$P\ddot{a}_{xy:\overline{10|}}^{00} + 0.75P\ddot{a}_{xy:\overline{10|}}^{01} = 673.63 + 361.98$$

$$\ddot{a}_{xy:\overline{10|}}^{00} = \int_0^{10} {}_tp_{xy}^{00} e^{-0.06t} dt = \int_0^{10} e^{-0.025t} e^{-0.06t} dt = 6.736295$$

$$\ddot{a}_{xy:\overline{10|}}^{01} = \int_0^{10} {}_tp_{xy}^{01} e^{-0.06t} dt = \int_0^{10} \frac{0.010}{0.013} \left( e^{-0.012t} - e^{-0.025t} \right) e^{-0.06t} dt = 0.30165075$$

$$P = \frac{673.63 + 361.98}{6.736295 + 0.75 \times 0.30165075} = 148.74$$

(d) ★★★★☆ Difficulty

If Molly is dead and Fred is alive, then the model is reduced to a simple alive-death model between state 1 and state 3 where the force of mortality is $\mu = 0.012$.

$$
\begin{aligned}
{}_5V^{(1)} &= 100,000\int_0^5 0.012e^{-0.012t}e^{-0.06t}dt - 0.75 \times 148.74\int_0^5 e^{-0.012t}e^{-0.06t}dt \\
&= 5038.7279 - 468.41274 = 4570.3152
\end{aligned}
$$

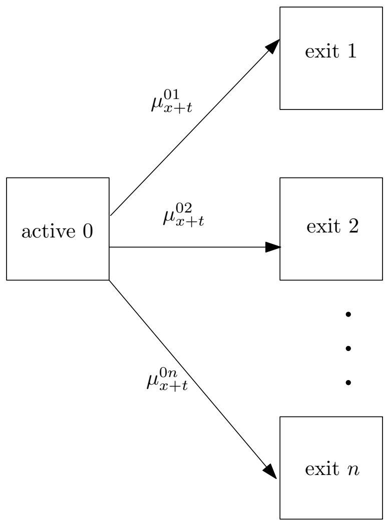
(e) ★★★☆☆ Difficulty

With a lower $\mu^{02}_{x+t:y+t}$, fewer insureds will go to State 2 and more insureds will go to State 1 and 3, causing EPV of the death benefit to go up.

WA Q6

(a) ★★★★★ Difficulty

In the general multiple decrement model, the active state 0 can be hit by $n$ mutually exclusive forces. Other than the active state 0, all the other states (state 1, state 2, ..., state $n$) are absorbing states.




$$
\begin{cases}
\dfrac{\mathrm{d}}{\mathrm{d}t}{}_tp_x^{00} = -{}_tp_x^{00}\left(\mu^{01}_{x+t} + \mu^{02}_{x+t} + \ldots + \mu^{0n}_{x+t}\right) \\
\dfrac{\mathrm{d}}{\mathrm{d}t}{}_tp_x^{0j} = {}_tp_x^{00}\mu^{0j}_{x+t} \text{ for } j \neq 0 \\
\dfrac{\mathrm{d}}{\mathrm{d}t}{}_tp_x^{jk} = 0 \text{ for } j \neq 0, k \neq 0, j \neq k
\end{cases}
$$

Using special notations for the general multiple state model, we can rewrite the above equations as:

$$
\begin{cases}
\dfrac{\mathrm{d}}{\mathrm{d}t}{}_tp_x^{(\tau)} = -{}_tp_x^{(\tau)}\left(\mu^{(1)}_{x+t} + \mu^{(2)}_{x+t} + \ldots + \mu^{(n)}_{x+t}\right) \\
\dfrac{\mathrm{d}}{\mathrm{d}t}{}_tq_x^{(j)} = {}_tp_x^{(\tau)}\mu^{(j)}_{x+t} \text{ for } j \neq 0
\end{cases}
$$

Note that the general multiple decrement model doesn't have any notation for ${}_tp_x^{jk}$ where $j \neq 0, k \neq 0, j \neq k$.

(b)(i) ★★★☆☆ Difficulty

$$
{}_2p_{60}^{(\tau)} = \big(1 - (0.05 + 0.10 + 0.08)\big)\big(1 - (0.00 + 0.14 + 0.12)\big) = (1 - 0.23)(1 - 0.26) = 0.5698
$$

(b)(ii) ★★★★★ Difficulty

To solve this part and part (c), you'll need to have deep understanding of the multiple decrement model. Just memorizing the formula won't work.

During Year 1,

- Force 1 is a point decrement occurring exactly at $t = 0.25$.
- Force 2 is UDD (in the multiple state model context).
- Force 3 is active during $[t = 0.5, t = 1]$.

Consider what happens during Year 1. Of the starting population ${}_0p_{60}^{(\tau)} = 1$ at $t = 0$,

- 0.05 of them are hit by Force 1 at $t = 0.25$.
- ${}_{0.8}q_{60}^{(2)} = 0.8 \times 0.1$ of them are hit by Force 2 during the interval $[t = 0, t = 0.8]$

- ${}_{0.8}q_{60}^{(3)} = 2(0.8-0.5)0.08$ of them are hit by Force 3 during the interval $[t=0.5, t=0.8]$

$$ {}_{0.8}p_{60}^{(\tau)} = 1 - \big(0.05 + 0.8\times 0.1 + 2(0.8-0.5)0.08\big) = 0.822 $$

$(c)(ii)$ ★★★★★ Difficulty

$$ {}_{t}p_{60}^{(\tau)} = \begin{cases} 1-0.1t & \text{if } 0 \le t < 0.25 \text{ (Force 2 working)} \\ 1-0.1\times 0.25 - 0.05 = 0.925 & \text{if } t = 0.25 \text{ (Force 1 working)} \\ 0.925-(t-0.25)0.1 & \text{if } 0.25 \le t \le 0.5 \text{ (Force 2 working)} \\ 0.925-(t-0.25)0.1 - 2(t-0.5)0.08 & \text{if } 0.5 \le t \le 1 \text{ (Force 2 and 3 working)} \end{cases} $$

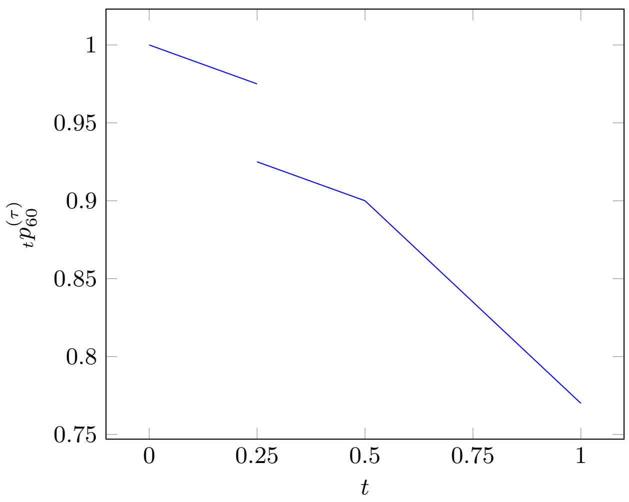
Let's verify. In Equation ${}_{t}p_{60}^{(\tau)} = 0.925-(t-0.25)0.1-2(t-0.5)0.08$, set $t=1$:

$$ {}_{1}p_{60}^{(\tau)} = 0.9-(1-0.5)0.1-2(1-0.5)0.08 = 0.77 $$

From the table:

| $x$ | $q_x^{(1)}$ | $q_x^{(2)}$ | $q_x^{(3)}$ |
|---|---|---|---|
| 60 | 0.05 | 0.10 | 0.08 |

$$ {}_{1}p_{60}^{(\tau)} = 1-(0.05+0.10+0.08) = 0.77 \text{ OK!} $$




$(d)(ii)$ ★★★★☆ Difficulty

We can start from the Kolmogorove forward equation:

$$ \frac{\mathrm{d}}{\mathrm{d}t}{}_{t}p_x^{0j} = {}_{t}p_x^{00}\mu_{x+t}^{0j} \text{ for } j\neq 0 $$

$$ \Rightarrow \quad \mu_{x+t}^{0j} = \frac{\frac{\mathrm{d}}{\mathrm{d}t}{}_{t}p_x^{0j}}{{}_{t}p_x^{00}} \text{ for } j\neq 0 $$

Translate into the notation in the multiple decrement model:

$$ \mu_{x+t}^{(j)} = \frac{\frac{\mathrm{d}}{\mathrm{d}t}{}_{t}q_x^{(j)}}{{}_{t}p_x^{(\tau)}} \text{ for } j\neq 0 $$

$$ \mu_{60+0.8}^{(2)} = \left(\frac{\frac{\mathrm{d}}{\mathrm{d}t}{}_{t}q_{60}^{(2)}}{{}_{t}p_{60}^{(\tau)}}\right)_{t=0.8} $$

$$ {}_{0.8}p_{60}^{(\tau)} = 0.925-(0.8-0.25)0.1-2(0.8-0.5)0.08 = 0.822 $$

$$ \frac{\mathrm{d}}{\mathrm{d}t}{}_{t}q_{60}^{(2)} = q_{60}^{(2)} = 0.1 \text{ for } 0\le t<1 $$

$$ \Rightarrow \quad \mu_{60+0.8}^{(2)} = \frac{0.1}{0.822} = 0.1216545 $$

$$ \frac{\mathrm{d}}{\mathrm{d}t}{}_{t}q_{60}^{(3)} = 2q_{60}^{(3)} = 2\times 0.08 \text{ for } 0.5\le t<1 $$

$$ \Rightarrow \quad \mu_{60+0.8}^{(3)} = \frac{2\times 0.08}{0.822} = 0.1946472 $$

## Chapter 2

# About the author

I was born in China and received a bachelor's degree in physics at Zhengzhou University and masters of law from Beijing Law School. I was a law school lecturer and a part time attorney in China. After I immigrated to U.S. in 1996, my law degree was no longer useful and I needed to re-invent myself. I didn't know what to do. One day I learned of the actuarial profession while working for an IT department of an insurance company. What attracted me about the actuarial profession was, embarrassingly, the exam raise. I had two young boys at that time and we lived in a run-down apartment. Money was tight. I was surprised to hear that you get a raise just by passing a math exam. I didn't think I would have any problems passing actuarial exams. I studied quantum physics in college and how hard can an actuarial exam be. However, when I discovered the actuarial profession, I hadn't touched calculus for 10 years. I borrowed books from a library, re-taught myself calculus and statistics in 2 months, and got a 9 in Exam P. I applied for an actuary job and was accepted.

Currently, I'm an ASA and an associate actuary at Milliman Buffalo Grove office at Chicago. Our company address is `http://us.milliman.com/`. My major expertise is life insurance experience studies (mortality and lapse studies), Monte Carlo simulations, and big data. I'm an expert in SAS. You can find my book *Beginning SAS Programming* at amazon. My email is `Yufeng.Guo@milliman.com` or `yufeng.guo.actuary@gmail.com`.

Here are sample chapters of my MLC study manual for Spring 2018: `http://deeperunderstandingfastercalc.com/how2Solve/MLCspring2018Sample.pdf`.