Chapter 1

Guo's Solution to Spring 2018 MLC MC Q1-Q20

For my bio and my LTAM study guide for Fall 2018, please refer to the last page.

MC Q1 ★★★★☆ Difficulty

$$p_{[x]} = p_{x-5}$$
$$p_{[x-1]+1} = p_{x-3} = p_{(x-1)-2} \quad \Rightarrow p_{[x]+1} = p_{x-2}$$
$$p_{[x-2]+2} = p_{x-1} = p_{(x-2)+1} \quad \Rightarrow p_{[x]+2} = p_{x+1}$$

$$\begin{aligned} {}_4p_{[45]} &= p_{[45]} \times p_{[45]+1} \times p_{[45]+2} \times p_{[45]+3} \\ &= p_{45-5} \times p_{45-2} \times p_{45+1} \times p_{48} \\ &= p_{40} \times p_{43} \times p_{46} \times p_{48} \\ &= 0.997219 \times 0.996557 \times 0.995686 \times 0.994956 = 0.98450735 \end{aligned}$$
$${}_4q_{[45]} = 1 - 0.98450735 = 0.01549265 \quad \text{ANSWER: } D$$

MC Q2 ★★★☆☆ Difficulty

${}_{0.2}p_{50.6} = 0.9$. Set $\ell_{50.6} = 1$. Then $\ell_{50.8} = 0.9$

$$\begin{cases} \ell_{50.6} = \ell_{50} - 0.6d_{50} = 1 \\ \ell_{50.8} = \ell_{50} - 0.8d_{50} = 0.9 \end{cases}$$
$$\Rightarrow \quad d_{50} = 0.5, \ell_{50} = 1.3$$
$$\ell_{51} = \ell_{50} - d_{50} = 1.3 - 0.5 = 0.8$$
$$q_{50} = \frac{d_{50}}{\ell_{50}} = \frac{0.5}{1.3} = 0.38461538 \quad \text{ANSWER: } E$$

MC Q3 ★★★☆☆ Difficulty

Let $x = y = 65$

$$F\left(\ddot{a}_x + \ddot{a}_y - \ddot{a}_{xy}\right) = 100,000 {}_5p_x {}_5p_y v^5$$
$$F\left(\ddot{a}_{65} + \ddot{a}_{65} - \ddot{a}_{65:65}\right) = 100,000 {}_5p_{65} {}_5p_{65} v^5 = \left({}_5E_{65}\right)^2 (1+i)^5$$
$$F\left(9.8969 + 9.8969 - 7.8552\right) = 100,000 \times 0.656225^2 \times 1.06^5$$
$$F = 4827.0463 \quad \text{ANSWER: } B$$

MC Q4 ★★★★☆ Difficulty

$$d^{(12)} = 12\left(1 - 1.05^{-1/12}\right) = 0.048691112$$
$$\ddot{a}_x^{(m)} = \frac{1 - A_x^{(m)}}{d^{(m)}} \quad \text{Exact formula; UDD not required}$$
$$\ddot{a}_{50}^{(12)} = \frac{1 - A_{50}^{(12)}}{d^{(12)}} = \frac{1 - 0.189}{0.048691112} = 16.656017$$
$$\ddot{a}_{65}^{(12)} = \frac{1 - A_{65}^{(12)}}{d^{(12)}} = \frac{1 - 0.354}{0.048691112} = 13.267308$$
$$\ddot{a}_{50:\overline{15}|}^{(12)} = \ddot{a}_{50}^{(12)} - {}_{15}E_{50}\ddot{a}_{65}^{(12)} = 16.656017 - 0.462 \times 13.267308 = 10.526521 \quad \text{ANSWER: } D$$

MC Q5 ★★★★★ Difficulty

- The interest rate in Year 1 is 4%
- The interest rate in Year 2 is 4.5%
- The interest rate in Year 3 is 5%

- The interest rate in Year 4 is 5.5%
- The interest rate in Year 5, Year 6, ... is 6%

$$
\begin{aligned}
APV = &\frac{\ell_{83}}{\ell_{80}} \times \frac{1}{1.04 \times 1.045 \times 1.05} \\
&+\frac{\ell_{84}}{\ell_{80}} \times \frac{1}{1.04 \times 1.045 \times 1.05 \times 1.055} \\
&+\frac{\ell_{85}}{\ell_{80}} \times \frac{1}{1.04 \times 1.045 \times 1.05 \times 1.055 \times 1.06} \\
&+\frac{\ell_{86}}{\ell_{80}} \times \frac{1}{1.04 \times 1.045 \times 1.05 \times 1.055 \times 1.06^2} + \dots
\end{aligned}
$$

The first term in the right hand side is:

$$\frac{2,970,496}{3,914,365} \times \frac{1}{1.04 \times 1.045 \times 1.05} = 0.66501084$$

Let $v = 1.06^{-1}$. The sum of the remaining terms in the right hand side is:

$$
\begin{aligned}
&\frac{\ell_{84} + \ell_{85}v + \ell_{86}v^2 + \dots}{\ell_{80}} \times \frac{1}{1.04 \times 1.045 \times 1.05 \times 1.055} \\
&= \frac{\ell_{84}}{\ell_{80}} \times \frac{\ell_{84} + \ell_{85}v + \ell_{86}v^2 + \dots}{\ell_{84}} \times \frac{1}{1.04 \times 1.045 \times 1.05 \times 1.055} \\
&= \frac{\ell_{84}}{\ell_{80}} \times \ddot{a}_{84} \times \frac{1}{1.04 \times 1.045 \times 1.05 \times 1.055} \\
&= \frac{2,660,734}{3,914,365} \times 4.92824 \times \frac{1}{1.04 \times 1.045 \times 1.05 \times 1.055} = 2.7825348
\end{aligned}
$$

$$100(0.66501084 + 2.7825348) = 344.75456 \quad \text{ANSWER: } C$$

MC Q6 ★★★★★ Difficulty

Let $x = 40$. The loss-at-issue random variable is:

$$L = \begin{cases} 100,000v^{T(x)} - 3,144 & \text{if } T(x) \le 10 \\ -3,144 & \text{if } T(x) > 10 \end{cases}$$

L is a decreasing function of $T(x)$.

$$100,000 \times 1.04^{-T(x)} - 3,144 = 75,000 \quad \Rightarrow T(x) = -\frac{\ln \frac{75000 + 3144}{100000}}{\ln 1.04} = 6.2879251$$

$$P(L \ge 75,000) = P\left(T(40) \le 6.2879251\right) = 1 - {}_{6.2879251}p_{40}$$

$$
\begin{aligned}
{}_{6.2879251}p_{40} &= \frac{\ell_{46.28792514}}{\ell_{40}} = \frac{(1 - 0.2879251)\ell_{46} + 0.2879251\ell_{47}}{\ell_{40}} \\
&= \frac{(1 - 0.2879251) \times 9,127,426 + 0.2879251 \times 9,088,049}{9,313,166} = 0.97883882
\end{aligned}
$$

$$1 - 0.97883882 = 0.02116118 \quad \text{Answer: } E$$

MC Q7 ★★★★☆ Difficulty

$$G\ddot{a}_{45:\overline{20}|} = 0.3G + 0.05G\ddot{a}_{45:\overline{10}|} + 0.05G\ddot{a}_{45:\overline{20}|} + 150,000\,{}_{20}E_{45}\ddot{a}_{65}$$

$$G(6.25 + 0.3 \times 6) = 0.3G + 0.05G \times 6.25 + 0.05G(6.25 + 0.3 \times 6) + 150,000 \times 0.3 \times 0.27 \times 7.4$$

$$G = 12780.384 \quad \text{Answer: } C$$

MC Q8 ★★★★☆ Difficulty

Let $x = 75$. Due to symmetry, APV of 1 paying continuously on the death of Jenn if she dies second is equal to APV of 1 paying continuously on the death of Dave if he dies second. Each APV is equal to $0.5\overline{A}_{\overline{xx}}$.

$$P\overline{a}_{xx} = 0.5\overline{A}_{\overline{xx}} = 100,000 \times 0.5(\overline{A}_x + \overline{A}_x - \overline{A}_{xx})$$

$$P \times \frac{1 - \overline{A}_{xx}}{\delta} = 100,000 \times 0.5(\overline{A}_x + \overline{A}_x - \overline{A}_{xx})$$

$$P \times \frac{1-0.7228}{\ln 1.06} = 100,000 \times 0.5(0.6091 + 0.6091 - 0.7228)$$

$$P = 5206.7852 \quad \text{Answer: } B$$

MC Q9 ★★★☆☆ Difficulty

$$0.15 = \frac{10,000,000}{1,000P\ddot{a}_{45}}$$

$$0.15 = \frac{10,000,000}{1,000P \times 14.11209} \Rightarrow P = 4724.0817 \quad \text{Answer: } B$$

MC Q10 ★★★★☆ Difficulty

$$P\overline{a}_x^{00} = 50,000\overline{A}_x^{01} + 50,000\overline{A}_x^{03} + 100,000\overline{A}_x^{02}$$
$$10.989P = 50,000 \times 0.39 + 50,000 \times 0.28 + 100,000 \times 0.181$$
$$P = 4695.6047 \quad \text{Answer: } B$$

MC Q11 ★★★★☆ Difficulty

To simplify notation, we'll use T to represent the number of premiums paid (e.g. $T = K_x + 1$).

$$L = 250,000v^T - 5,000\ddot{a}_{\overline{T}|} = 250,000v^T - 5,000\frac{1-v^T}{d} = \left(250,000 + \frac{5,000}{d}\right)v^T - \frac{5,000}{d}$$

Set $L = 0$:

$$\left(250,000 + \frac{5,000}{d}\right)1.06^{-T} - \frac{5,000}{d} = 0$$

$$\left(250,000 + \frac{5,000}{1-1.06^{-1}}\right)1.06^{-T} - \frac{5,000}{1-1.06^{-1}} = 0$$

$$(338333.33)1.06^{-T} - 88333.333 = 0, T = -\frac{\ln\dfrac{88333.333}{338333.33}}{\ln 1.06} = 23.046838$$

The first premium is paid at time zero. If the insured is alive at $t = 23$, he will have paid at least 24 premiums, causing $L < 0$ (negative loss).

$$_{23}p_{55} = \frac{\ell_{78}}{\ell_{55}} = \frac{4,530,360}{8,640,861} = 0.52429497 \quad \text{Answer: } A$$

MC Q12 ★★★☆☆ Difficulty

$$12P\ddot{a}^{(12)}_{40:\overline{20}|} = 1,000,000A_{40:\overline{20}|}$$

$$\ddot{a}^{(12)}_{40:\overline{20}|} = \alpha(12)\ddot{a}_{40:\overline{20}|} - \beta(12)\left(1 - {}_{20}E_{40}\right)$$
$$= 1.000281 \times 11.76126 - 0.468120\left(1 - 0.274137\right) = 11.424774$$
$$12P \times 11.424774 = 1,000,000 \times 0.334269, P = 2438.1883 \quad \text{Answer: } C$$

MC Q13 ★★★☆☆ Difficulty

$$\left(66,600 + 10,000 \times 0.95 - 120 - (500,000 - AV_{11})0.015/1.04\right)1.04 = AV_{11}, \quad AV_{11} = 72,608.325$$
$$ADB = 500,000 - 72,608.325 = 427,391.68 \quad \text{Answer: } A$$

MC Q14 ★★★★☆ Difficulty

$$_{10.4}V \times 1.06^{0.6} = 1,000,000 \times {}_{0.6}q_{45.4} + {}_{11}V \times {}_{0.6}p_{45.4}$$

$$_{11}V = 1,000,000A^{\;1}_{46:\overline{9}|} - 3,489\ddot{a}_{46:\overline{9}|} = 1,000,000 \times 0.039132 - 3,489 \times 7.07590 = 14,444.185$$

$$e^{-\mu} = p_{45}, \quad {}_{0.6}p_{45.4} = e^{-0.6\mu} = (p_{45})^{0.6} = 0.996003^{0.6}$$

$$_{10.4}V \times 1.06^{0.6} = 1,000,000\left(1 - 0.996003^{0.6}\right) + 14,444.185 \times 0.996003^{0.6}$$
$$_{10.4}V = 16,232.105 \quad \text{Answer: } E$$

MC Q15 ★★★☆☆ Difficulty

$$2700 \times 0.95 + 18700 \times 0.04 - (100000 \times 1.025 - 18700)0.013 = 2,223.6 \quad \text{Answer: } B$$

MC Q16 ★★★★☆ Difficulty

$$_{20}V = 1000\left(1 - \frac{\ddot{a}_{60}}{\ddot{a}_{40}}\right)$$

$$ {}_{20}V^F = 1000\left(1 - \frac{\ddot{a}_{60}}{\ddot{a}_{41}}\right) $$

$$ {}_{20}V - {}_{20}V^F = 1000\ddot{a}_{60}\left(\frac{1}{\ddot{a}_{41}} - \frac{1}{\ddot{a}_{40}}\right) $$

$$ = 1000 \times 9.72\left(\frac{1}{13.04} - \frac{1}{13.25}\right) = 11.813867 \quad \text{Answer: } A $$

MC Q17 ★★★★☆ Difficulty

$$ (16074 + 900 \times 0.9)1.05^{0.5} = 100,000{}_{0.5}q_{55.5} + {}_{11}V \times {}_{0.5}q_{55.5} $$

$$ \ell_{55.5} = 0.5(\ell_{55} + \ell_{56}) = 0.5(8,640,861 + 8,563,435) = 8,602,148 $$

$$ {}_{0.5}q_{55.5} = \frac{8,563,435}{8,602,148} = 0.9955 $$

$$ (16074 + 900 \times 0.9)1.05^{0.5} = 100,000(1 - 0.9955) + {}_{11}V \times 0.9955, \quad {}_{11}V = 16,927.124 \quad \text{Answer: } A $$

MC Q18 ★★★★☆ Difficulty

Since $\mu^{02} = \mu^{12}$, in terms of the death rate, we can combine the healthy and the sick into one node and simplify the original diagram into an alive-death model.

$$ {}_5p_x^{02} = 1 - e^{-0.002\times 5} = 0.0099501663 \quad \text{Answer: } B $$

Alternative method.

$$ \frac{d}{dt}{}_tp_x^{02} = \left({}_tp_x^{00} + {}_tp_x^{01}\right)0.002 = \left(1 - {}_tp_x^{02}\right)0.002 $$

Set $1 - {}_tp_x^{02} = y(t)$.

$$ -\frac{d}{dt}y = 0.002y, y = Ae^{-0.002t}, {}_tp_x^{02} = 1 - Ae^{-0.002t} $$

Using the initial condition ${}_0p_x^{02} = 0$, we have $A = 1$ and ${}_tp_x^{02} = 1 - e^{-0.002t}$.

MC Q19 ★★★★☆ Difficulty

$$ P\left(\ddot{a}_{\overline{10}|} + {}_{10}E_{65}\ddot{a}_{75}\right) = 30 \times 0.02 \times \frac{100000}{3}\left(1 + \frac{1}{1.03} + \frac{1}{1.03^2}\right)\ddot{a}_{65} $$

$$ P\left(\frac{1 - 1.06^{-10}}{1 - 1.06^{-1}} + 0.399941 \times 7.21702\right) = 30 \times 0.02 \times \frac{100000}{3}\left(1 + \frac{1}{1.03} + \frac{1}{1.03^2}\right)9.89693 $$

$$ 10.688074P = 576688.11 $$

$$ P = 53,956.221 \quad \text{Answer: } E $$

MC Q20 ★★★★☆ Difficulty

$$ {}_0V = 0, \quad \text{because YOS=0} $$

$$ \text{For } {}_1V, \quad YOS = 1 $$

$$ {}_1E_{50}{}_1V = \frac{10000 \times d_{64}^{(d)} \times 1.05^{-(64.5-50)} + 15000 \times d_{65}(d) \times 1.05^{-(65.5-50)}}{\ell_{50}} $$

$$ {}_1E_{50}{}_1V = \frac{10000 \times 257 \times 1.05^{-14.5} + 15000 \times 204 \times 1.05^{-15.5}}{29919} = 90.35 $$

$$ {}_0V + NC = {}_1E_{50}{}_1V, \quad 0 + NC = 90.35, \quad NC = 90.35 \quad \text{Answer: } D $$

Since salary is irrelevant to the death benefit, there's no difference between PUC and TUC.

Chapter 2

About the author

I was born in China and received a bachelor's degree in physics at Zhengzhou University and masters of law from Beijing Law School. I was a law school lecturer and a part time attorney in China. After I immigrated to U.S. in 1996, my law degree was no longer useful and I needed to re-invent myself. I didn't know what to do. One day I learned of the actuarial profession while working for an IT department of an insurance company. What attracted me about the actuarial profession was, embarrassingly, the exam raise. I had two young boys at that time and we lived in a run-down apartment. Money was tight. I was surprised to hear that you get a raise just by passing a math exam. I didn't think I would have any problems passing actuarial exams. I studied quantum physics in college and how hard can an actuarial exam be. However, when I discovered the actuarial profession, I hadn't touched calculus for 10 years. I borrowed books from a library, re-taught myself calculus and statistics in 2 months, and got a 9 in Exam P. I applied for an actuary job and was accepted.

Currently, I'm an ASA and an associate actuary at Milliman Chicago office. My major expertise is life insurance experience studies (mortality and lapse studies), Monte Carlo simulations, and big data. I'm an expert in SAS. You can find my book *Beginning SAS Programming* at amazon. My email is `Yufeng.Guo@milliman.com` or `yufeng.guo.actuary@gmail.com`.

Here are sample chapters of my MLC study manual for Spring 2018: `http://deeperunderstandingfastercalc.com/how2Solve/MLCspring2018Sample.pdf`. I'm currently updating it for Fall 2018. I expect to release my Fall 2018 LTAM study guide on June 15, 2018.