Contents

L.M.1 ★★★☆☆ Difficulty

$$ {}_{12}p_x = {}_{10}p_x \times {}_2p_{x+10} = {}_{10}p_x \left(1 - {}_2q_{x+10}\right) = 0.8\left(1 - \frac{100}{600-200}\right) = 0.6 $$

L.M.2 ★★★☆☆ Difficulty

This is similar to Loss Model Example 12.3

$$ S_n(x) = 1 - F_n(x) $$

In this problem, $n = 1000$.

$$ F_n(1) = \frac{200}{1000} = 0.2, \quad S_n(1) = 0.8 $$

$$ F_n(1) \sim \text{binomial}(n = 1000, p = 0.2) $$

$$ \widehat{Var}\left[S_n(1)\right] = \widehat{Var}\left[F_n(1)\right] = \frac{pq}{n} = \frac{0.8 \times 0.2}{1000} = 0.012649 $$

$$ 0.8 \pm 1.96 \times 0.012649 = 0.8 \pm 0.02479 = [0.775,\ 0.825] $$

L.M.3 ★★★☆☆ Difficulty

During the interval $(a, b]$, if death occurs exactly at b and not at any other times, under the Nelson-Aalen method, the accumulative hazard rate over $(a, b]$ is

$$ \hat{H}_{a \to b} = \frac{\ell_{b-} - \ell_{b+}}{\ell_{b-}} $$

The conditional probability of being alive at $b+$ given being alive at $a+$ is:

$$ P(a^+ \to b^+) = {}_{b-a}p_a = \exp\left(-\hat{H}_{a \to b}\right) = \exp\left(-\frac{\ell_{b-} - \ell_{b+}}{\ell_{b-}}\right) $$

where $+$ means immediately after and $-$ means immediately before.

- ℓ_{b-} is the number of people alive immediately before b
- ℓ_{b+} is the number of people alive immediately after b

$$ \hat{H}_{0 \to 1} = \frac{\ell_{1-} - \ell_{1+}}{\ell_{1-}} = \frac{1}{100-10} = \frac{1}{90} $$

$$ \hat{H}_{1 \to 1.5} = \frac{\ell_{1.5-} - \ell_{1.5+}}{\ell_{1.5-}} = \frac{3}{90-9} = \frac{3}{81} $$

$$ \hat{H}_{0 \to 1.5} = \frac{1}{90} + \frac{3}{81} = 0.048148 $$

$$ \hat{S}(1.5) = {}_{1.5}p_0 = {}_1p_0 \times {}_{0.5}p_1 = \exp\left(-\frac{1}{90}\right)\exp\left(-\frac{3}{81}\right) = e^{-0.048148} = 0.95299273 $$

L.M.4 ★★★★☆ Difficulty

During the interval $(a, b]$, if death occurs exactly at b and not at any other times, under the Kaplan-Meier method, the conditional probability of being alive at $b+$ given being alive at $a+$ is:

$$ P(a^+ \to b^+) = {}_{b-a}p_a = \frac{\ell_{b+}}{\ell_{b-}} $$

Make sure you understand the notation:

- i is the event counter (so first event, second event, and so on)
- y_i is the even time
- s_i is number of deaths at time y_i
- b_i is the number of lapses at time y_i

We can construct the risk set r_i, which is the number of people alive immediately before y_i. The number of people alive at $t = 5^-$ is the number of the deaths and the the number of the lapses that have occurred on/after $t = 5$:

$$ r_1 = \ell_{5-} = \sum_{i=1}^{5} (s_i + b_i) = 60 $$

Similarly, the number of people alive at $t=8^-$ is the number of the deaths and the the number of the lapses that have occurred on/after $t=8$:

$$r_2 = \ell_{8^-} = \sum_{i=2}^{5}(s_i+b_i) = 48$$

$$r_3 = \ell_{13^-} = \sum_{i=3}^{5}(s_i+b_i) = 35$$

$$r_4 = \ell_{16^-} = \sum_{i=4}^{5}(s_i+b_i) = 21$$

$$r_5 = \ell_{21^-} = \sum_{i=5}^{5}(s_i+b_i) = 10$$

$$P(0^+ \to 5^+) = {}_5p_0 = \frac{\ell_{5+}}{\ell_{5-}} = \frac{60-5}{60}$$

ℓ_{5+} includes the $b_1=7$ lapses that occurred at $t=5$. For each lapse at $t=5$, we assume that the lapsed subject under our study is alive at $t=5+$ (e.g. the earliest time death time for a lapse at $t=5$ is a few seconds after $t=5$).

$$P(5^+ \to 8^+) = {}_3p_5 = \frac{\ell_{8+}}{\ell_{8-}} = \frac{48-6}{48}$$

$$P(8^+ \to 13^+) = {}_5p_8 = \frac{\ell_{13+}}{\ell_{13-}} = \frac{35-7}{35}$$

$$P(13^+ \to 16^+) = {}_3p_{13} = \frac{\ell_{16+}}{\ell_{16-}} = \frac{21-6}{21}$$

$$P(16^+ \to 21^+) = {}_5p_{16} = \frac{\ell_{21+}}{\ell_{21-}} = \frac{10-6}{10}$$

$$\hat{S}(21) = {}_{21}p_0 = \frac{60-5}{60} \times \frac{48-6}{48} \times \frac{35-7}{35} \times \frac{21-6}{21} \times \frac{10-6}{10} = 0.18333333$$

Unfortunately, there's no intuitive explanation for $\widehat{Var}[S_n(y)]$. The good news is that don't have to memorize it. The LTAM table provides the Greenwood's approximation formula.

$$\widehat{Var}[S_n(21)] = \left[\hat{S}(21)\right]^2 \sum_{y_i \le 21} \frac{s_i}{r_i\,(r_i - s_i)}$$

$$= 0.18333333^2 \left(\frac{5}{60(60-5)} + \frac{6}{48(48-6)} + \frac{7}{35(35-7)} + \frac{6}{21(21-6)} + \frac{10}{10(10-6)} \right) = 0.0060729 = 0.07793^2$$

$$0.18333333 \pm 1.282 \times 0.07793 = [0.08342, 0.28324]$$

L.M.5 ★★★☆☆ Difficulty

This is similar to Loss Model Example 12.4.

Probability of exit by $t=10$:

$$F_{100}(10) = \frac{28+19}{100} = 0.47$$

Probability of exit by $t=20$:

$$F_{100}(20) = \frac{28+19+15}{100} = 0.62$$

We find $F_{100}(12)$ by linearly interpolating $[10, 0.47]$ and $[20, 0.62]$.

$$F_{100}(12) = 0.47 + \frac{0.62-0.47}{20-10} \times (12-10) = 0.5$$

L.M.6 ★★★☆☆ Difficulty

This is similar to Loss Model Example 12.28. Total time spent in state 0:

$$T_0 = 3 + 0.35 + (1-0.75) + 0.45 = 4.05$$

Number of transitions from state 0 to state 1:

$$d_{01} = 2$$

$$\widehat{Var}\left(\hat{\mu}_x^{01}\right) = \frac{d_{01}}{T_0^2} = \frac{2}{4.05^2} = 0.34919^2$$

S1.1. ★★★★☆ Difficulty

- Policy coverage: $[1/1/2018, 12/31/2025]$
- Alice's sickness: $[7/1/2018, 12/1/2018]$ and $[3/1/2019, 11/1/2019]$.

- Waiting period (e.g. elimination period): 2 months
- Off period: 4 months. If two sickness incidents are 0 month to 4 months apart, then the second incident is treated as continuation of the first incident and hence the second incident will trigger benefit payments without having to start a new waiting period.

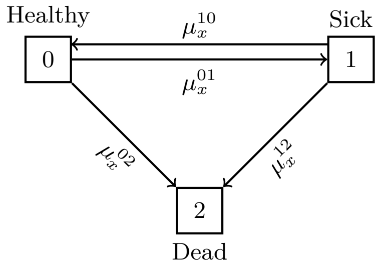
There are 5 months in the first sickness period and 8 months in the second sickness period. Since the two sickness incidents are 4 months apart, there won't be a new waiting period for the second sickness. The total number of the benefit periods is:

$$5-2 \text{ waiting period } +8=11 \text{ months}$$

Alice will receive a check on each of the following dates: 9/1/2018, 10/1/2018, 11/1/2018, and 12/1/2018, totaling 4 checks. On 12/1/2018 immediately after receiving the check, she recovers from her sickness and goes back to work. She becomes ill again on 3/1/2019. Since the second illness and the first illness are 4 months apart, which doesn't exceed the off period, Alice will receive checks without staring over a new waiting period. She will receives a check on each of the following dates: 4/1/2019, 5/1/2019, ..., and 11/1/2019, totaling 7 checks.

Alice will receive a total of 11 checks.

S2.1 ★★★★☆ Difficulty



$$\ddot{a}^{01}_{50:\overline{10}|} = {}_0p^{01}_{50} + {}_1p^{01}_{50}v + {}_2p^{01}_{50}v^2 + \ldots + {}_9p^{01}_{50}v^9$$
$$a^{01}_{50:\overline{10}|} = {}_1p^{01}_{50}v + {}_2p^{01}_{50}v^2 + \ldots + {}_9p^{01}_{50}v^9 + {}_{10}p^{01}_{50}v^{10}$$

$${}_0p^{01}_{50} = 0 \quad \Rightarrow \quad a^{01}_{50:\overline{10}|} = \ddot{a}^{01}_{50:\overline{10}|} + {}_{10}p^{01}_{50}v^{10}$$

We can find ${}_{10}p^{01}_{50}$ in the LTAM table. We need to find $\ddot{a}^{01}_{50:\overline{10}|}$.

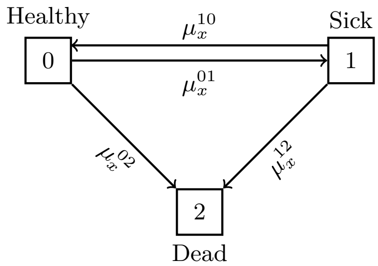
$$\ddot{a}^{01}_{50} = \ddot{a}^{01}_{50:\overline{10}|} + {}_{10}p^{00}_{50}v^{10}\ddot{a}^{01}_{60} + {}_{10}p^{01}_{50}v^{10}\ddot{a}^{11}_{60}$$

$$1.9618 = \ddot{a}^{01}_{50:\overline{10}|} + 0.83936 \times 1.05^{-10} \times 2.6283 + 0.06554 \times 1.05^{-10} \times 10.7144$$

$$\ddot{a}^{01}_{50:\overline{10}|} = 0.176349$$

$$\Rightarrow \quad a^{01}_{50:\overline{10}|} = 0.176349 + 0.06554 \times 1.05^{-10} = 0.21658$$

S3.1 ★★★★☆ Difficulty



$$\left({}_{10}V^{(0)} + 0.95P\right)1.06 = {}_1p^{00}_{60}\,{}_{11}V^{(0)} + {}_1p^{01}_{60}\left(30000 \times 1.05 + {}_{11}V^{(1)}\right)$$
$$\left({}_{10}V^{(1)}\right)1.06 = {}_1p^{10}_{60}\,{}_{11}V^{(1)} + {}_1p^{11}_{60}\left(30000 \times 1.05 + {}_{11}V^{(1)}\right)$$

In each equation above, the left hand side is funding and the righthand side is future liabilities. For an insured who is either healthy or sick at $t=10$, the funding should exactly pay the future liabilities at $t=11$.

$$(5946 + 0.95 \times 2360)1.06 = 0.97026\,{}_{11}V^{(0)} + 0.01467\left(30000 \times 1.05 + {}_{11}V^{(1)}\right)$$
$$(200640)1.06 = 0.00313\,{}_{11}V^{(1)} + 0.97590\left(30000 \times 1.05 + {}_{11}V^{(1)}\right)$$

$$ {}_{11}V^{(0)} = 5650.5525 $$
$$ {}_{11}V^{(1)} = 186412.4 $$

$$ \left({}_{10}V^{(0)} + 0.95P\right) 1.05 = {}_1p_{60}^{00}\, {}_{11}V^{(0)} + {}_1p_{60}^{01} \left(30000 \times 1.05 + {}_{11}V^{(1)}\right) $$
$$ \left({}_{10}V^{(1)}\right) 1.05 = {}_1p_{60}^{10}\, {}_{11}V^{(1)} + {}_1p_{60}^{11} \left(30000 \times 1.05 + {}_{11}V^{(1)}\right) $$

To understand this equation, imagine that the original insurer sold you the DII policies 10 years after inception (e.g. at $t = 10$). For each healthy insured now age 60, the seller gives you ${}_{10}V^{(0)} = 5,946$ dollars. The original owner also warns you that for each sick insured now age 60 the APV of the future claims liability of 200, 640.

Consider what happens from $t = 10$ to $t = 11$ for each healthy insured at $t = 10$.

- At $t = 10$, each healthy insured will pay a net premium $0.95P$. Your total asset is ${}_{10}V^{(0)} + 0.95P$.
- At $t = 11$, if the insured is healthy, you need to set side a fund equal ${}_{11}V^{(0)}$ to help pay his future liability (because he can be sick at any time).
- At $t = 11$, if the insured is sick, you immediately incur cost of 30000×1.05. In addition, you need to set aside a fund equal to ${}_{11}V^{(1)}$ to help pay this newly sick's future claims.

Set your total asset plus earned interest in one year to the total liability and you'll get the first equation.

S4.1 ★★★★☆ Difficulty

- For someone aged x today, let $q(x + n, n)$ represent his n year forward mortality rate.
- For someone aged $x + n$ today, let $q(x + n, 0)$ represent his spot mortality rate (e.g. the same age mortality rate today).

The n year forward mortality rate is the same age mortality rate today multiplied by n reduction factors.

$$ q(x+n,n) = q(x+n,0)\left(1-\varphi(x+n,1)\right)\left(1-\varphi(x+n,2)\right)\ldots\left(1-\varphi(x+n,n)\right) $$

- zero-year forward mortality rate is $q(0, 0) = 0.4$
- 1-year forward mortality rate is $q(1,1) = q(1,0)\left(1-\varphi(1,1)\right) = 0.5(1-0.08)$
- 2-year forward mortality rate is $q(2,2) = q(2,0)\left(1-\varphi(2,1)\right)\left(1-\varphi(2,2)\right) = 0.6(1-0.06)(1-0.04)$

$$ {}_3p_0 = (1-0.4)\times(1-0.5(1-0.08))\times(1-0.6(1-0.06)(1-0.04)) = 0.14857344 $$

S4.2 ★★★★☆ Difficulty

The information (i) "no cohort effect" tells you to use the age-based cubic spline, note the cohort-based cubic spline.

Let t represent the number of years after 2017. The mortality improvement function and its derivative are:

$$ \begin{cases} \varphi(x, 2017+t) = C_a(x,t) = at^3 + bt^2 + ct + d \\ \left[\dfrac{d}{dt}\varphi(x, 2017+t)\right]_t = 3at^2 + 2bt + c \end{cases} $$

The 4 boundary conditions:

$$ \begin{cases} \varphi(35, 2017+0) = 0.037 \quad \text{start point} \\ \left[\dfrac{d}{dt}\varphi(35, 2017+t)\right]_{t=0} = \varphi(35,2017+0) - \varphi(35,2017-1) = 0.037 - 0.035 \quad \text{derivative at start} \\ \varphi(35, 2017+10) = 0.015 \quad \text{end point} \\ \left[\dfrac{d}{dt}\varphi(35, 2027+t)\right]_{t=10} = \varphi(35,2017+11) - \varphi(35,2017+10) = 0 \quad \text{derivative at end} \end{cases} $$

Now we have:

$$ \begin{cases} d = 0.037 \\ c = 0.002 \\ a \times 10^3 + b \times 10^2 + c \times 10 + d = 0.015 \\ 3\left(10^2\right)a + 2(10)b + c = 0 \end{cases} $$

$$ \begin{cases} a = 0.000064 \\ b = -0.00106 \\ c = 0.002 \\ d = 0.037 \end{cases} $$

$$\Rightarrow \varphi(35, 2022) = C_a(35, 2022 - 2017) = C_a(35, 5) = 0.000064 \times 5^3 - 0.00106 \times 5^2 + 0.002 \times 5 + 0.037 = 00.0285$$

Note that there's an inconsistency between $\left[\frac{d}{dt}\varphi(35, 2017+t)\right]_{t=0}$ and $\left[\frac{d}{dt}\varphi(35, 2017+t)\right]_{t=10}$. To be consistent, the fourth boundary condition should be written as:

$$\left[\frac{d}{dt}\varphi(35, 2017+t)\right]_{t=10} = \varphi(35, 2017+10) - \varphi(35, 2019+9)$$

However, this inconsistency doesn't matter much because smoothing is art, not rocket science.

S4.3 S4.4 S4.5 S4.6
★★★★★ Difficulty
This is similar to Mary Hardy Study Note Example 4.5.

$$\ln m(60, 2020) = \alpha_{60} + \beta_{60} K_{2020}$$
$$K_{2020} = K_{2019} + c + \sigma_K Z_{t_1}$$
$$K_{2019} = K_{2018} + c + \sigma_K Z_{t_2}$$
$$\Rightarrow \quad K_{2020} = K_{2018} + 2c + \sigma_K (Z_{t_1} + Z_{t_2}) = -3 + 2(-0.05) + 0.9\,(Z_{t_1} + Z_{t_2})$$

$$\ln m(60, 2020) = -4 + 0.25\left(-3 + 2(-0.05) + 0.9\,(Z_{t_1} + Z_{t_2})\right) = -4.775 + 0.225\,(Z_{t_1} + Z_{t_2})$$

Z_{t_1} and Z_{t_2} are *iid* standard normal random variable.

$$E\left[\ln m(60, 2020)\right] = -4.775 + 0.225(0+0)$$
$$Var\left[\ln m(60, 2020)\right] = 0.225^2(1+1)$$

$$\ln m(50, 2018) \sim N(\mu = -4.775, \text{Var} = 0.225^2 \times 2)$$

> If $\ln Y \sim N\left(\mu, \sigma^2\right)$, then $E\left[Y^n\right] = e^{n\mu + 0.5n^2\sigma^2}$

$$E\left[m(60, 2020)\right] = e^{1(-4.775)+0.5\times1\times0.225^2\times2} = e^{-4.724375} = 0.00887626$$
$$E\left[m(60, 2020)^2\right] = e^{2(-4.775)+0.5\times2^2\times0.225^2\times2} = e^{-9.3475}$$
$$Var\left[m(60, 2020)\right] = e^{-9.3475} - e^{-2\times4.724375} = 8.3951159 \times 10^{-6}$$
$$\sigma\left[m(60, 2020)\right] = \sqrt{8.3951159 \times 10^{-6}} = 2.8974326 \times 10^{-3} = 0.002987$$

$m(60, 2020) = \exp\left(\ln m(60, 2020)\right)$ is an increasing function of $\ln m(60, 2020)$. As a result, 95-th percentile of $m(60, 2020)$ maps to 95-th percentile of $\ln m(60, 2020)$.

$$Q_{95\%}[\ln m(60, 2020)] = -4.775 + \Phi^{-1}(0.95)\sqrt{0.225^2 \times 2} = -4.775 + 1.645\sqrt{0.225^2 \times 2} = -4.2515642$$
$$Q_{95\%}[m(60, 2020)] = e^{-4.2515642} = 0.014242$$

> Under UDD between integral ages:
>
> $$m(x) = \frac{{}_1q_x}{\int_0^1 {}_rp_x dr} = \frac{q_x}{{}_{0.5}p_x} = \frac{q_x}{1 - 0.5q_x}$$
>
> Solving the equation above, we get:
>
> $$q_x = \frac{m_x}{1 + 0.5m_x}, \quad p_x = \frac{1 - 0.5m_x}{1 + 0.5m_x}$$

$p(60, 2020)$ is a decreasing function of $m(60, 2020)$. Consequently, 95-th percentile of $p(60, 2020)$ maps to 5-th percentile of $m(60, 2020)$, which maps to 5-th percentile of $\ln m(60, 2020)$.

$$Q_{5\%}[\ln m(60, 2020)] = -4.775 + \Phi^{-1}(0.05)\sqrt{0.225^2 \times 2} = -4.775 - 1.645\sqrt{0.225^2 \times 2} = -5.2984358$$
$$Q_{5\%}[m(60, 2020)] = e^{-5.2984358} = 0.005$$
$$Q_{95\%}[p(60, 2020)] = \frac{1 - 0.5 \times 0.005}{1 + 0.5 \times 0.005} = 0.99501$$

S5.1

★★★★☆ Difficulty

alive-death model $\bar{a}_x = \dfrac{1-\overline{A}_x}{\delta}$

general multiple state model where d is the death state $\displaystyle\sum_{j\neq d} \bar{a}_x^{0j} = \dfrac{1-\overline{A}_x^{0d}}{\delta}$

$$\sum_{j\neq 3} \bar{a}_x^{0j} = 0.52 + 3.24 + 5.6 = \frac{1-\overline{A}_x^{03}}{0.04}$$

$$\overline{A}_x^{03} = 0.6256$$

S6.1
★★★★☆ Difficulty

$$\ddot{a}_B(xr,t) = 1 + v\,{}_1p_{xr}\frac{B(xr+1,t+1)}{B(xr,t)} + v^2\,{}_2p_{xr}\frac{B(xr+2,t+2)}{B(xr,t)} + \ldots = \ddot{a}_{xr|i^*} = 1 + v_{i^*}\,{}_1p_{xr} + v_{i^*}^2\,{}_2p_{xr} + \ldots$$

$$\text{where } 1+i^* = \frac{1+i}{c(1+j)}$$

note that $\dfrac{B(xr+1,t+1)}{B(xr,t)}$ term is absored into i^*

$$\ddot{a}_B(63,2) = \ddot{a}_{63|i^*} = \ddot{a}_{63:\overline{2}|i^*} + v_{i^*}^2\,{}_2p_{63}\ddot{a}_{65|i^*}$$

$$1+i^* = \frac{1+0.06}{1.03(1+0.04)}$$

$$v_{i^*} = \frac{1.03(1+0.04)}{1+0.06} = 1.010566$$

$$\ddot{a}_B(63,2) = 1 + v_{i^*}\,{}_1p_{63} + v_{i^*}^2\,{}_2p_{63}\ddot{a}_{65|i^*}$$

$$= 1 + 1.010566(1-0.00473) + 1.010566^2(1-0.00473)(1-0.005288)26.708 = 29.008599$$

S6.2
★★★★★ Difficulty
From the standard ultimate life table,

$$\ell_{50} = 98,576.4$$

$$\ell_{60} = 96,634.1, \quad r_{60} = 0.5\ell_{60} \text{ will retire immediately at age 60}$$

Out of the remainder population $0.5\ell_{60}$ at age 60, the number of survivors at age 61 is:

$$0.5\ell_{60} \times {}_1p_{60} = 0.5\ell_{60} \times \frac{\ell_{61}}{\ell_{60}} = 0.5\ell_{60} = 0.5 \times 96,305.8$$

$$r_{61} = 0.5\ell_{60} \text{ will retire immediately at age 61}$$

Let y represent the actual retirement age. Let y^* represent the adjusted retirement age after considering mid-year exit. For example, for an actual retirement age $y = 61$, $y^* = 60.5$. This problem doesn't have mid-year exit so $y = y^*$. However, we'll still use y^* so our notation will be consistent. $x = 50$ is the age at the valuation date and $e = 30$ is the entry age (e.g. age when first hired).

Even though the problem doesn't ask for ${}_0V$, we'll calculate it any way. Later we'll reuse the ${}_0V$ formula to get NC.

$${}_0V = \frac{B(60,0)(1+j)^{60-50}}{\ell_{50}} v_i^{60-50} \sum_{y^*=60}^{61} r_y\,\ddot{a}_{y^*|i^*}\,v_{i^*}^{y^*-60} \times \frac{x-e}{y^*-e}$$

$$i = 0.05 \text{ actual interest rate}$$

$$i^* = \frac{1+i}{c(1+j)} - 1 = \frac{1+0.05}{1.0194(1+0.03)} - 1 \approx 0 \text{ adjusted interest rate}$$

$${}_0V = \frac{5,000 \times 1.03^{10}}{\ell_{50}} \times 1.05^{-10}\left(r_{60}\ddot{a}_{60|i^*}\,v_{i^*}^{60-60} \times \frac{50-30}{60-30} + r_{61}\,\ddot{a}_{61|i^*}\,v_{i^*}^{61-60} \times \frac{50-30}{61-30}\right)$$

$$i^* = 0 \quad \Rightarrow a_{x|i^*} = e_x, \quad \ddot{a}_{x|i^*} = 1 + e_x, \quad v_{i^*}^{61-60} = 1$$

$$ {}_0V = \frac{5000 \times 1.03^{10}}{98576.4} \times 1.05^{-10} \left(0.5 \times 96634.1 \times 1(1+26.71) \times \frac{50-30}{60-30} + 0.5 \times 96305.8 \times 1(1+25.8) \times \frac{50-30}{61-30} \right) = 72194.6 $$

To find NC_x, we'll recalculate ${}_0V$ by adding one more year of service.

$$ {}_0V^* = \frac{5000 \times 1.03^{10}}{98576.4} \times 1.05^{-10} \left(0.5 \times 96634.1(1+26.71) \times \frac{50+1-30}{60-30} + 0.5 \times 96305.8(1+25.8) \times \frac{50+1-30}{61-30} \right) $$

$$ \begin{aligned} NC_{50} &= {}_0V^* - {}_0V \\ &= \frac{5000 \times 1.03^{10}}{98576.4} \times 1.05^{-10} \left(0.5 \times 96634.1(1+26.71) \times \frac{1}{60-30} + 0.5 \times 96305.8(1+25.8) \times \frac{1}{61-30} \right) \\ &= 3609.73 \end{aligned} $$