Contents

Purpose of this document. To illustrate how to tackle WA questions when you have only partial knowledge.

For WA question, doing numerical calculation is not important. What's important is to explain, systematically, how to solve a problem. This way, the grader knows that you understand the core concepts.

3 examples from Sample LTAM WA questions are used.

WA 1.

(a) PV R.V. DB:

$$Z = \begin{cases} 0 & \text{if } K_{40}+1 = 1, 2, \dots, 20 \\ 20,000 v^{K_{40}+1} & \text{if } K_{40}+1 = 21, 22, \dots \end{cases}$$

$$\text{EPV DB} = E(Z)$$

(b) Equivalence: PV Prem=PV DB

$$P\ddot{a}_{40}^{(12)} = E(Z)$$

$$\ddot{a}_{40}^{(12)} = \ddot{a}_{40} - \frac{11}{24}$$

P is annualized prem.

(c) If you are landlord. You have 2 options:

(i) collect one big rent monthly in advance

(ii) collect smaller rent everyday

Annualized rent in (ii) higher than in (i) because

- If tenant die or run away midway, no rent in (ii) but there's rent in (i)
- one big rent in (i) earns more interest than deposit daily rent in a bank

Same for prem. prem go up.

(d)

- Before. Pay prem monthly in advance while Ben alive
- After. Pay prem monthly in advance while Ben alive but waive prem after Anne dies

More conditions in after than before so life annuity factor in after lower than before. But DB same.

So prem after > Prem before.

Prem incr = PVDB/new smaller annuity factor - PVDB/old bigger annuity factor

- In (a), if you know the answer, you don't need to define Z; you can directly write $20,000 {}_{20}E_{40} A_{60}$. However, I don't know the answer and I don't want to leave it blank. So I defined Z.
- I used lists instead of writing full sentences.
- I ignored grammar (if tenant die or run away)
- I used many abbreviations. DB (death benefit), prem (premium), R.V. (random variable), b/c (because). Abbreviations are OK if they are not cryptic.
- I used an analogy to explain why premium will go up. If you can't explain something rigorously, explain by example.

WA 2. (a)

$$\frac{d}{dt} {}_tp_0^{00} = ?$$

$$\frac{d}{dt} {}_tp_0^{22} = 0 \quad \text{once in state always in state 2}$$

${}_tp_x^{ij}$: prob that insured in state j at age $x+t$ given in state i at age x.

initial conditions:

$${}_0p_0^{00} = 1$$

$${}_0p_0^{01} = 0$$

$${}_0p_0^{02} = 0$$

$(b)(i)$

$$f(x+h) \approx f(x) + hf'(x)$$

$${}_{0.5}p_0^{01} \approx {}_0p_0^{01} + 0.5 \left[\frac{d}{dt} {}_tp_0^{00} \right]_{t=0}$$

$(b)(ii)$ Use $f(x+h) \approx f(x) + hf'(x)$ repeatedly to get ${}_1p_0^{02}$.

$${}_{0.5}p_0^{02} \approx {}_0p_0^{02} + 0.5 \left[\frac{d}{dt} {}_tp_0^{02} \right]_{t=0}$$

$$ {}_1p_0^{02} \approx {}_{0.5}p_0^{02} + 0.5 \left[\frac{d}{dt} {}_tp_0^{02} \right]_{t=0.5} $$

$(c)(i)$ 1-year term insurance with DB=1000 payable end of 6-month of death with death being getting state 2.

$$ \text{APV DB} = 1000A_x^{(2)} = 1000 \left(v \times {}_{0.5}p_0^{02} + v^2 \times {}_1p_0^{02} \right) $$

$$ v = 1.04^{-1} $$

$(c)(ii)$

$$ P\ddot{a}_0^{(2)} = \text{APV DB} $$
$$ \ddot{a}_0^{(2)} = 1 + {}_{0.5}p_0^{00} v $$

(d) User smaller h such as 0.01.

> I forgot the formula for $\frac{d}{dt} {}_tp_x^{ij}$, but I still want to write something meaningful down to earn partial credit.
> At least I know that $\frac{d}{dt} {}_tp_0^{22} = 0$.
> I wrote down $f(x+h) \approx f(x) + hf'(x)$ because this is the foundation for the Euler's method.

WA 3. (a)

$$ S = X_1 + X_2 + \ldots + X_{1000} $$

$X_i = 1$ if i-th person dead at $t = 1$ and $X_i = 1$ if alive. X_i's are iid. $S \sim$ binomial $(n = 1,000, p = 0.2)$

$$ E(S) = np, \quad Var(S) = npq $$

(b)

$$ U = Y_1 + Y_2 + \ldots + Y_{1000} $$

$Y_i = 1$ if i-th person dead at $t = 1$ and $Y_i = 0$ if alive. Y_i's are not iid.

- With prob 0.8, U is $U_1 \sim$ binomial $(n = 1,000, p = 0.2)$
- With prob 0.2, U is $U_2 \sim$ binomial $(n = 1,000, p = 0.05)$

$$ E(U) = 0.8E(U_1) + 0.2E(U_2) = 0.8(1000 \times 0.2 + 1000 \times 005) $$
$$ E(U^2) = 0.8E(U_1^2) + 0.2E(U_2^2) $$
$$ E(U_1^2) = E^2(U_1) + Var(U_1) = (1000 \times 0.2)^2 + 1000 \times 0.2 \times 0.8 $$
$$ E(U_2^2) = E^2(U_2) + Var(U_2) = (1000 \times 0.05)^2 + 1000 \times 0.05 \times 0.95 $$
$$ Var(U) = E(U^2) - E^2(U) $$

(c)

$$ W = Z_1 + Z_2 + \ldots + Z_{1000} $$

$Z_i = 1$ if i-th person dead at $t = 1$ and $Z_i = 0$ if alive. Z_i's are iid.
Z_i is mixture of 2 Bernoulli R.Vs:

- Z_i^1 has $p = 0.2$. prob 0.8
- Z_i^2 has $p = 0.05$. prob 0.8.

$$ E(Z_i) = 0.8E(Z_i^1) + 0.2E(Z_i^2) = 0.8 \times 0.2 + 0.2 \times 0.05 $$
$$ E(Z_i^2) = 0.8 \left(E(Z_i^1) \right)^2 + 0.2 \left(E(Z_i^2) \right)^2 = \ldots $$
$$ Var(W) = 1000Var(Z_i) = 1000 \left(E(Z_i^2 - E^2(Z_i) \right) $$

> For this problem, I ran out of time. However, I outlined the step to calculating the variance.