SMALL & MIGHTY

MLC: PENSION

Small & Mighty: MLC Pension

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Yufeng Guo

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Preface

pension problems make or break your exam

Pension plans and retirement benefits account for 10% to 20% of the exam points. In the Spring 2017 MLC exam, there are 3 multiple choice questions (Q18, Q19, and Q20, totaling 6 points) and one written answer question (Q6, totaling 9 points). In this book, you'll learn how to solve a series of difficult problems:

- How to calculate the actuarial liability and the normal cost under PUC and TUC actuarial cost methods
- How to tackle mid-year exit problems
- How to calculate the EPV of the accrued withdrawal benefit, with or without COLA
- How to set DC contribution level to meet the target replacement ratio
- How to calculate the replacement ratio under different retirement ages
- How to use a service table
- How to calculate the actuarial liability and the normal cost where some members are active, some are in deferment, and others are collecting pension checks

motivating example

Not sure whether you need this book? Here's a problem for you. If you can solve it, you don't need this book.

Example 0.0.1

Data about a pension plan and its member Jeff:

- Current age: 57
- YOS (years of service): 20
- Normal retirement age: 65
- Pension benefit: 2% of final salary for each year of service payable monthly in advance for life. The pension benefit is guaranteed for 5 years.
- Final salary: salary during the year before retirement or withdrawal
- If member withdraws from the pension plan, the pension benefit is deferred till the normal retirement age
- Members can withdraw from the pension plan at any time before reaching age 60
- Salary during the year preceding the current age: \$100,000
- Annual salary growth: 4%
- Service Table: Illustrative Service Table
- Post-withdrawal survival: Illustrative Life Table and UDD
- $i = 0.06$
- Selective actuarial values: $j = 2.912621\%$

y	$A^{(12)}_{1\ \overline{y:65-y}\rceil i}$	$A^{(12)}_{1\ \overline{y:65-y}\rceil j}$
57.0	0.08699	0.09847
57.5	0.08463	0.09505
58.0	0.08217	0.09154
58.5	0.07920	0.08755
59.0	0.07609	0.08344
59.5	0.07241	0.07880
60.0	0.06856	0.07402
60.5	0.06406	0.06864
61.0	0.05935	0.06310
61.5	0.05392	0.05689
62.0	0.04822	0.05049
62.5	0.04170	0.04333
63.0	0.03486	0.03595
63.5	0.02708	0.02772
64.0	0.01892	0.01922
64.5	0.00969	0.00978
65.0	0.00000	0.00000

(a) (1 point) Show that $\ddot{a}^{(12)}_{65:\overline{5}|} = 9.66579$

(b) (2 points) Show that the EPV of the accrued withdrawal pension with no COLA is approximately \$4,000. You should calculate the EPV to the nearest of 10.

(c) (2 points) Show that the EPV of the accrued withdrawal pension with a COLA in deferment of 3% per year is approximately \$5,000. You should calculate the EPV to the nearest of 10.

(d) (1 point) Show that $A^{(12)}_{\substack{1\\57.5:\overline{1}|}} = 0.01075$

(e) (2 points) On death during deferment, a lump sum benefit of five times the accrued annual pension, with no COLA, is paid at the end of the month of death. Estimate the EPV of this benefit.

(f) (2 points) On death during deferment, a lump sum benefit of five times the accrued annual pension, with a COLA of 3% per year, is paid at the end of the month of death. Estimate the EPV of this benefit.

Solution to this problem is in Chapter 5.

why Small & Mighty series

Chances are that you already have your favorite study manuals and authors. No need to switch manuals or authors. Keep using your favorite study manuals but use Small & Mighty to beef up your knowledge of some topics in MLC.

solve any pension problems SOA throws at you

Pension problems show up every year and they often are difficult. See for yourself whether you can solve Spring 2017 MLC WA Q6. It's one of the hardest problem in the exam.

Many candidates are afraid of pension problems. They would rather solve ten general life contingency problems than one pension problem. There are just so many twists and turns in a pension problem. Mid-year exit? Withdrawal? To name a few.

In this book, you'll learn how to tackle thorny pension problems, either multiple choice or written answer. My goal is to help you become a pension expert so you can solve any pension problems SOA throws at you.

Frankly, it took me two months to figure out the ins and outs of the AMLCR Chapter 10. If you don't have two months to learn pension, please follow me and save yourself some exam woes. If you can improve your score by 5 points after reading this book, it'll be time and money well spent.

Chapter 1

Pension math: accrued liability and normal cost

1.1 actuarial cost methods

The present value for all the plan benefits during a member's career from entry into the pension to the retirement age is called the present value of benefits (PVB) or the present value of future benefits (PVFB). PVB or PVFB includes the benefits earned during past employment and the benefits to be earned during future employment.

Ideally, when a member retires, a fund equal to the PVB was already set up to pay for his retirement benefits. Pension sponsors use actuarial cost methods to allocate the PVB to the member's career. This helps the pension sponsor fund the PVB while the PVB is being earned. If assumptions are correct, the pension sponsor could theoretically set aside the right amount of money in a plan today to cover payments from the plan, including payments for past employment and those for future employment.

The portion of the PVB or PVFB allocated to past employment is called actuarial liability (AL) or actuarial accrued liability (AAL); the AMLCR represents this by ${}_tV$. The portion of the PVB or PVFB allocated to the current year of service is the normal cost (NC); the AMLCR represents this by C_t. Finally, the portion of the PVB or PVFB allocated to future employment is called the present value of future normal costs (PVFNC). PVFNC is not discussed by the AMLCR. Different cost methods calculate these 3 components differently.

The AMLCR focuses on two cost methods: PUC (project unit credit) method and TUC (traditional unit credit method).

1.2 PUC, TUC, AL, NC

Under PUC and TUC, the actuarial liability is

$$ {}_0V = AL_x = {}_{r-x}E_x \, B_x \, \ddot{a}_r^{(12)} = {}_{r-x}E_x \, \alpha S_{fin}(x-e) \, \ddot{a}_r^{(12)} $$
$$ B_x = \alpha S_{fin}(x-e) $$

- r is the normal retirement age.
- x is the pension participant's attained age on the valuation date.
- e is the entry age into the pension plan.
- $YOS = x - e$ is the years of service.
- AL_x is the EPV of the accrued pension benefit as of the valuation date. AL_x is most commonly used symbol. The AMLCR textbook uses ${}_0V$ to mean AL_x.
- B_x is the accrued benefit as of the valuation date. This is the annual pension benefit amount to be received after retirement due to past service.
- α is the accrual rate. α is typically between 0.01 and 0.02.
- S_{fin} is the average salary in a specified period before retirement used for calculating B_x. For example, S_{fin} can be the average salary of the last 3 years before retirement.
- To reduce calculation, most exam problems set $S_{fin} = S_{r-1}$, that is, the final average salary is equal to the final 1-year salary before retirement. This is called the final salary plan.
- S_{r-1} is the salary earned between the age $r-1$ and r. And generally, S_x is the salary earned between age x and $x+1$. Be careful because it's very easy to make an off-by-one error.
- However, as of the valuation date, we don't know the salaries to be earned during the n-years before retirement. Two major methods to estimate S_{fin}:
  - PUC (traditional unit credit method). This is the easier method of the two conceptually and it agrees to our intuition. We assume that an employee's salary grows each year till retirement. Two major methods can be used to model the salary growth: (1) a salary scale table such as the AMLCR Table 10.1, and (2) a constant salary growth rate.
  - TUC (traditional unit credit method). This is tricky. Remember the following key points:
    * TUC doesn't mean a constant salary throughout as you might think.
    * For a final salary plan, to calculate B_x and AL_x, set $S_{Fin} = S_{x-1}$, that is, to use the salary during the year immediately before the valuation date as the final salary. This is why. If the employer decides to stop offering the pension plan immediately after the valuation date, then S_{x-1} will be the salary used for calculate B_x and AL_x.

* Similarly, for a final salary plan, to calculate B_{x+1} and AL_{x+1}, set $S_{Fin} = S_x$. Since the valuation date for B_{x+1} and AL_{x+1} is one year after the valuation date for B_x and AL_x, S_x is used to calculate B_{x+1} and AL_{x+1}.
* $NC_x = v\ {}_1p_x^{00}\ AL_{x+1} - AL_x$ + EPV of benefits for mid-year exits. As a result, NC_x reflects the salary growth.

Paradoxically, for a final salary plan under TUC, even though TUC doesn't project salary growth, NC_x funds the salary growth because (1) AL_x and NC_x are calculated on the same valuation date, (2) NC_x depends on AL_{x+1}, and (3) AL_{x+1} uses S_x.

PUC or TUC: $\quad {}_0V = AL_x = {}_{r-x}E_x\ B_x\ \ddot{a}_r^{(12)} = {}_{r-x}E_x\ \alpha S_{fin}(x-e)\ \ddot{a}_r^{(12)}$

Big Idea

If a pension plan doesn't pay any death benefit or withdrawal benefit, then the pension benefit is a deferred whole life annuity due where the annual payment is $(x-e)\,\alpha\,S_{fin}$ payable typically monthly in advance starting from age r.

The annual payment of the pension benefit is far more complex than that of a typical annuity. As a result, the setup of a pension problem can be overwhelming. Instead of one sentence like "For a life annuity of 1,000 on (x)," you'll see paragraphs after paragraphs of details. SOA has to give you all the inputs: e, r, x, α, current salary, future salaries. Fear not! Just extract critical inputs for the AL_x and NC_x formulas (to be explained later).

1.3 accrued benefit recursive formula

Big Idea

$$AL_x + NC_x = EPV \text{ benefits due to termination in the next year} + v\ {}_1p_x^{00}\ AL_{x+1}$$

This formula is similar to the recursive policy value formula.

- AL_x (or ${}_tV$) is similar to the policy value
- NC_x (or C_t) is similar to the premium.
- *EPV of benefits for mid year exit* is similar to the EPV of the next year's death benefit

The beginning AL_x plus new funding source NC_x will, on average, exactly fund the exit benefit for those who leave the pension plan during the next year and the ending AL_{x+1} for those who remain in the pension plan.

By the way, if we assume that terminations occur in the mid year, the above equation becomes what's in the AMLCR textbook:

$${}_tV + C_t = EPV \text{ benefits for mid year exit} + v\ {}_1p_x^{00}\ {}_{t+1}V$$

1.4 illustrative problems

Example 1.4.1

You are given the following data about a pension participant Jeff and the pension plan:

- Age as of the valuation date: $x = 40$
- Entry age into the pension plan: $e = 30$
- Normal retirement age: $r = 65$
- Salary during the year prior to the valuation date: 60,000
- Accrual rate: $\alpha = 2\%$
- S_{fin}: final salary
- Pension benefit: a life annuity payable monthly in advance
- Benefit due on death in service: zero

Assumptions:

- No exits other than by death before normal retirement age
- Interest rate: 6%
- Salary increase: at $g = 3\%$ per year (projected unit credit)
- Mortality before and after retirement: Illustrative Life Table

Calculate the accrued actuarial liability and the normal contribution as of the valuation date using each of the following methods: PUC and TUC.